

**Cylindrical**  $\nabla \times \mathbf{A} = \left[ \frac{1}{s} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] \hat{\mathbf{s}} + \left[ \frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[ \frac{\partial (sA_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi} \right] \hat{\mathbf{z}}$       $\bar{\nabla} f = \hat{\mathbf{s}} \frac{\partial f}{\partial s} + \hat{\phi} \frac{1}{s} \frac{\partial f}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial f}{\partial z}$

$\nabla \cdot \mathbf{A} = \frac{1}{s} \frac{\partial (sA_s)}{\partial s} + \frac{1}{s} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$       $d\tau = s ds d\phi dz$       $da = s d\phi dz$       $d\mathbf{l} = ds \hat{\mathbf{s}} + s d\phi \hat{\phi} + dz \hat{\mathbf{z}}$

$\nabla^2 V = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial V}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$       $d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\theta} + r \sin\theta d\phi \hat{\phi}$      Polar (2d)  $da = r dr d\theta$

**Spherical**  $\bar{\nabla} = \hat{\mathbf{r}} \frac{\partial f}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial f}{\partial \theta} + \hat{\phi} \frac{1}{r \sin\theta} \frac{\partial f}{\partial \phi}$       $da = r^2 d\theta \sin\theta d\phi$       $d\tau = dr da = r^2 dr d\theta \sin\theta d\phi$

$\nabla \times \mathbf{A} = \frac{1}{r \sin\theta} \left[ \frac{\partial}{\partial \theta} \sin\theta A_\phi - \frac{\partial A_\theta}{\partial \phi} \right] \hat{\mathbf{r}} + \frac{1}{r} \left[ \frac{1}{\sin\theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial (rA_\phi)}{\partial r} \right] \hat{\theta} + \frac{1}{r} \left[ \frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \hat{\phi}$

$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta A_\theta) + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi}$

**Cartesian**  $\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$       $\bar{\nabla} f = \hat{\mathbf{x}} \frac{\partial f}{\partial x} + \hat{\mathbf{y}} \frac{\partial f}{\partial y} + \hat{\mathbf{z}} \frac{\partial f}{\partial z}$       $\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$       $d\tau = dx dy dz$

$d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}$      Laplacian  $\nabla^2 V = \nabla \cdot (\nabla V)$

**Electrostatics**  $\mathbf{E} = \frac{q}{4\pi\epsilon_0} \frac{\hat{\mathbf{r}}}{r^2}$       $\nabla^2 V = -\frac{\rho}{\epsilon_0}$       $\mathbf{E} = -\nabla V$       $\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\zeta}{\zeta^2} \rho(r') d\tau'$  where  $\zeta = \mathbf{r} - \mathbf{r}'$

$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\text{surface}} \frac{\sigma(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} da'$       $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\text{volume}} \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\tau'$       $V(r, \theta) = \sum_{l=0}^{\infty} \left( A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$

$\nabla^2 V = 0 \Rightarrow \frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} + \frac{1}{Z} \frac{d^2 Z}{dz^2} = 0$       $dq = \rho d\tau = \sigma da = \lambda dx$       $W = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 d\tau$

Gauss's Law:  $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$       $V(r_b) - V(r_a) = -\int_{r_a}^{r_b} \mathbf{E} \cdot d\mathbf{l}$       $E_{\perp}^{\text{above}} - E_{\perp}^{\text{below}} = \frac{\sigma}{\epsilon_0}$

**Magnetostatics**  $I = \lambda v$       $\mathbf{K} = \sigma \mathbf{v}$       $\mathbf{J} = \rho \mathbf{v}$       $\mathbf{K}_b = \mathbf{M} \times \mathbf{n}$       $\mathbf{J}_b = \text{curl}(\mathbf{M})$

$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I d\mathbf{l} \times \hat{\zeta}}{\zeta^2} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J} \times \hat{\zeta}}{\zeta^2} d\tau$  where  $\zeta = \mathbf{r} - \mathbf{r}'$       $\mathbf{B} = \nabla \times \mathbf{A}$       $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$       $\nabla \times \mathbf{H} = \mathbf{J}_{\text{free}}$

$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$       $I_{\text{enc}} = \int \mathbf{J} \cdot d\mathbf{a}$

$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \rightarrow \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$       $\nabla \cdot \mathbf{B} = 0$       $\mathbf{B}_{\text{dip}} = \frac{\mu_0 m}{4\pi r^3} (2 \cos\theta \hat{\mathbf{r}} + \sin\theta \hat{\theta})$

$\mathbf{F} = Q\mathbf{v} \times \mathbf{B}$       $d\mathbf{F} = I d\mathbf{l} \times \mathbf{B} = da \mathbf{K} \times \mathbf{B} = d\tau \mathbf{J} \times \mathbf{B}$

**Dipoles etc.**  $\mathbf{p} = \int_{\text{volume}} \mathbf{r}' \rho(\mathbf{r}') d\tau' = \sum_i \mathbf{r}_i q_i$       $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int_{\text{volume}} d\tau' (r')^n P_n(\cos(\theta')) \rho(\mathbf{r}')$       $V_{\text{dipole}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$

$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$       $\rho_b = -\nabla \cdot \mathbf{P}$       $\mathbf{N} = \mathbf{p} \times \mathbf{E}$  or  $\mathbf{N} = \mathbf{m} \times \mathbf{B}$ ,  $\mathbf{F} = \nabla(\mathbf{p} \cdot \mathbf{E})$  or  $\nabla(\mathbf{m} \cdot \mathbf{B})$

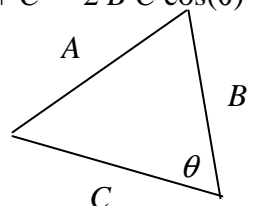
$\mathbf{E}_{\text{dip}} = \frac{p}{4\pi\epsilon_0 r^3} (2 \cos\theta \hat{\mathbf{r}} + \sin\theta \hat{\theta})$      Linear dielectric:  $\mathbf{P} = \chi_e \epsilon_0 \mathbf{E}$ , where  $\mathbf{p} = \mathbf{P} d\tau$

$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$       $\nabla \cdot \mathbf{D} = \rho_{\text{free}}$       $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$  (linear dielectrics)      $\mathbf{D} = \epsilon_0 (1 + \chi_e) \mathbf{E} = \epsilon \mathbf{E}$

**Theorems, constants, etc.**  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$       $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$       $A^2 = B^2 + C^2 - 2BC \cos(\theta)$

Stokes Theorem  $\rightarrow \int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$       $\int_{\text{volume}} \nabla \cdot \mathbf{A} d\tau = \oint_{\text{surface}} \mathbf{A} \cdot d\mathbf{a}$

$P_0(x) = 1$ ,  $P_1(x) = x$ ,  $P_2(x) = \frac{1}{2}(3x^2 - 1)$ , ...     Divergence Theorem



**Some integrals.**

$$\int_0^{\pi} d(\cos \theta) P_m(\cos \theta) P_l(\cos \theta) = \begin{cases} 0 & \text{if } m \neq l \\ \frac{2}{2m+1} & \text{if } m = l \end{cases}$$

$$\int \sin^3 x \, dx = \frac{-1}{3} \cos(x)(\sin^2 x + 2) \quad \int \sin^4 x = \frac{3x}{8} - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}} \quad \int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx = \begin{cases} 0 & \text{if } n \neq m \\ a/2 & \text{if } n = m \end{cases}$$

$$\int \sin(kx) dx = \frac{-1}{k} \cos(kx) \quad \int \frac{\sin(\theta) d\theta}{\sqrt{1 - \cos(\theta)}} = 2\sqrt{1 - \cos(\theta)} \quad \int \cos(kx) dx = \frac{1}{k} \sin(kx)$$