Statistical Mechanics / Phy4523
Formula Sheet as of 19-Apr-05

$R = 8.3145$ J/mol/K $N_{avogadro} = 6.0221 \times 10^{23}$ /mole 1 eV = 1.6022×10^{-19} Joules

$h = 6.6261 \times 10^{-34}$ J-s $k_B = 1.38065 \times 10^{-23}$ J/K = 8.6173×10^{-5} eV/K $\sigma = 5.670 \times 10^{-8}$ W/m^2 K^4

$c = 2.9979 \times 10^{8}$ m/s $dQ_{\text{rev}} = T\, dS_{\text{rev}}$ $dU = TdS - pdV$ $1/T = (\partial S/\partial U)_V$

$C_v = (\partial U/\partial T)_v = T(\partial S/\partial T)_V$ $H = U + pV$ $F = U - TS$ $G = U - TS + pV$

$S = k_B \log(W)$ (microcanonical) $S = -k_B \sum_i p_i \log(p_i)$ (canonical)

Stefan-Boltzmann Law $dQ/dt = A\sigma T^4$

Combinations of r objects out of n objects: ${}^nC_r = n!\,/\,r!\,(n-r)!$

Stirling's Approximation $\log(N!) \approx N\log(N) - N$ $N! = N(N-1)(N-2)(N-3)\ldots 1$

Maxwell Boltzmann Distribution (1-dimension) (Note A = normalization)

$$f(v_x)dv_x = A\exp(-mv_x^2/2k_BT)dv_x$$

Maxwell Boltzmann Distribution (3-dimensions) $f(v)d^3v = f(v_x)dv_x\, f(v_y)dv_y\, f(v_z)dv_z$

where $d^3v = dv_x dv_y dv_z = v^2 dv \sin(\theta) d\theta d\varphi$

Gaussian Integral $\sqrt{\pi} = \int_{-\infty}^{\infty} e^{-x^2} dx$

Gaussian Distribution $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-(x-\bar{x})^2/2\sigma^2)$

Density of states:

1d	$D(k)dk = (L/\pi)dk$
2d	$D(k)dk = (A/2\pi)\, k\, dk$
3d	$D(k)dk = (V/2\pi^2)\, k^2 dk$

Partition function $Z = \sum_i \exp(-E_i/k_BT)$ $F = -k_BT\log(Z)$ $S = -(\partial F/\partial T)_V$

Canonical Ensemble $p_i = \exp(-E_i/k_BT)/Z$ $U = \sum_i E_i p_i = k_BT^2 \left(\frac{\partial \log(Z)}{\partial T}\right)_V$

Sum of a geometric series: $\sum_{n=0}^{N} x^n = \frac{1-x^{N+1}}{1-x}$ If $N \rightarrow \infty$ and $x < 1$ then this simplifies to $1/(1-x)$

$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$ $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ $\tanh(x) = \sinh(x)/\cosh(x)$

Planck Formula $u(E)dE = D(E)dE \cdot \bar{n}(E) \cdot E$ where the energy of one photon in a mode of wavelength λ is $E = hc/\lambda = \hbar ck$ and the average number of photons in that mode is $\bar{n}(E) = Z^{-1}\sum_{n=0}^{\infty} n e^{nE/k_BT} = 1\big/(e^{E/k_BT} - 1)$

Grand Partition Function $\Xi = \sum_i e^{-(\varepsilon_i - \mu N_i)/k_BT}$ $p_i = e^{-(\varepsilon_i - \mu N_i)/k_BT}\big/\Xi$ $\Phi = -k_BT\log\Xi$

$N = -\left(\partial\Phi/\partial\mu\right)_{T,V} = k_BT\left(\partial\log\Xi/\partial\mu\right)_{T,V}$ ideal gas $\mu = k_BT\log(n/n_Q) + \Delta(\mathbf{r})$

Fermi-Dirac Distrib: $n(k) = \frac{1}{e^{(\varepsilon(k)-\mu)/k_BT} + 1}$ Bose-Einstein Distrib: $n(k) = \frac{1}{e^{(\varepsilon(k)-\mu)/k_BT} - 1}$