Tutorial on Quantum Turbulence

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Quantum Turbulence

An Introduction

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Classical and Quantum Turbulence

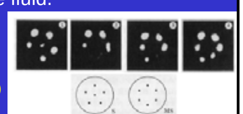
- Turbulence in ordinary liquids is governed by classical fluid mechanics.
- Flow of a superfluid is severely influenced by quantum effects.
- So what happens to turbulence in a superfluid:
  - can it exist?
  - if yes, how is it influenced by quantum effects?
- Today we will answer these questions with respect to the quantum fluids studied so far:

  ^{4}He, ^{3}He, and Cold Gases.

What is a superfluid?

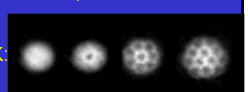
- Superfluid has some form of Bose condensation: the accumulation of a large fraction of the particles into a single quantum state. It is analogous to the accumulation of a large number of photons into a single quantum state in a laser, and it leads to a coherent particle field in the fluid.
- Examples:
  - Liquid ^{4}He below 2.2K →

    (single atoms undergo Bose condensation)



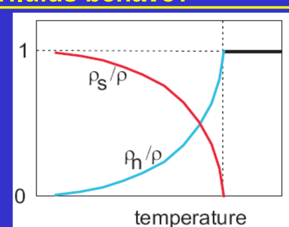
  - Liquid ^{3}He below about 2 mK

    (pairs of atoms undergo a form of Bose condensation)
  - Trapped atomic gases below 100 nK:



How do superfluids behave?

- ***Two fluids from one set of atoms***:



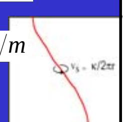
- **normal** (viscous) **component** ρ_n co-existing with
- inviscid **superfluid component** ρ_s
- separate velocity fields: $\mathbf{v}_s$ and $\mathbf{v}_n$

Superfluids are irrotational

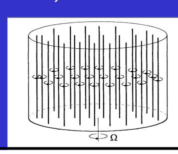
- Superflow connected with condensed particles
- Circulation of superfluid component in multiply-connected volume must be quantized:

  $\oint \mathbf{v}_s \cdot d\mathbf{r} = n(h/m)$ $\quad m = m_4$ or $2\mathrm{m}_3$

- Singly-quantized free vortex line: circulation $\kappa = h/m$ around a small hole (vortex core) radius ξ. (ξ~0.05nm in ^{4}He; ξ~80nm in ^{3}He-B)

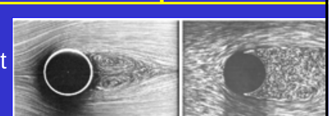


- Quantized vortex lines allow superfluid component to rotate with a containing vessel



Quantum turbulence in a superfluid

- Interesting questions:
  - How is typical turbulent motion affected by the quantization of circulation?



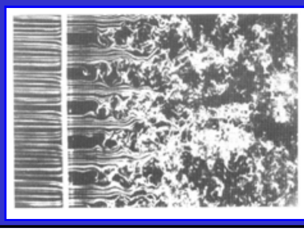
  - What happens to turbulence in an inviscid fluid, **when there is no dissipation**?

  - What happens to turbulence when we have two co-existing fluids-made of the same atoms?!
  - New effects, not seen in classical turbulence?
  - New types of turbulence?

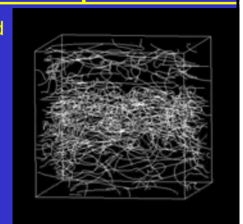
Quantum turbulence at zero temperature

- No normal fluid.
- But quantization of circulation

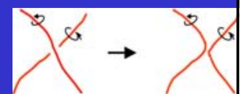
  and no viscous dissipation!
- Specifically: what happens to grid turbulence?



Rotational motion at finite temperature



- Only form of rotational motion allowed is the quantized vortex filament. Therefore turbulence **must** take the form of some tangle of these vortex filaments (simulation by Tsubota et al).
- What laws govern the time-evolution of this tangle?
  - Mostly classical: the vortices move with the local fluid velocity.
  - BUT one important effect is not described correctly by classical fluid mechanics:

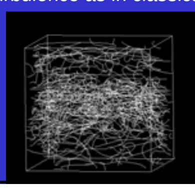


The Vortex Reconnection

Quantum grid turbulence at zero temperature

- **Experiments** (eg measurement of pressure fluctuations → energy spectrum, $E(k)$):
  - Experiments in ^{4}He down to fairly low temperatures (ρ_n/ρ~0.01)
  - Promising experiments in ^{3}He-B down to very low temperatures (ρ_n/ρ~0)
  - Experiments being prepared in ^{4}He at mK temperatures
  - No way yet to visualize quantum turbulence as in classical turbulence.
- **Simulations**

  For example: Evolution of vortex tangle at zero temperature (Tsubota *et al*)



Present understanding of Grid Turbulence in ^{4}He

- There is an important ***quantum length scale*** in quantum turbulence: ℓ, the spacing between quantized vortices.

- On scales >> ℓ, turbulence is essentially classical, with Kolmogorov energy spectrum: $E(k)=C\varepsilon^{2/3}k^{-5/3}$

Volumes containing many circulation quanta behave classically.

- On scales <~ ℓ, the vortex structure makes the turbulence non-classical.
- To have a Kolmogorov spectrum at small *k* we must have some form of dissipation at high *k*.

Dissipation in a quantum inviscid fluid at T=0?

Classical turbulence radiates sound.

Can this lead to dissipation in quantum turbulence?

- Sound can radiate from vortex tangle if regions of low density near vortex cores oscillate in position at a sufficiently high frequency.
- Smooth tangle with average vortex spacing ℓ oscillates in position with frequency of order $\omega_\ell = \kappa/\ell^2$

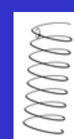
  Too small for efficient radiation of sound.
- Can the tangle evolve so that higher frequencies are produced?

Let us speculate →

High frequency motion in a vortex tangle

- Evolving turbulence moves turbulent energy to smaller length scales (on scales < ℓ) . Vortex close encounters lead to reconnections.

- Reconnection generates kinks on the two lines—like plucking a string → waves on the string. Waves on a vortex are helical Kelvin waves: dispersion relation



$$\tilde{\omega}(\tilde{k}) \approx \frac{\kappa \tilde{k}^2}{4\pi} \ln\left(\frac{1}{\tilde{k}\xi}\right)$$

- Sharp kinks → high $\tilde{k}$ → high frequency → efficient radiation of sound.

Kelvin-wave turbulence

- This Kelvin-wave turbulence is very interesting in itself and is the subject of much current research. Provided that weak-turbulence theory applies, there is probably a direct energy cascade with energy spectrum

$$\tilde{E}(\tilde{k}) \sim \rho^{0.8}\kappa^{1.4}\varepsilon^{0.2}\tilde{k}^{-1.4}$$

together with an inverse cascade that involves a flow of wave action to smaller wave numbers.

- But our understanding of these cascades and the way in which they are generated remains incomplete.
- Interesting theoretical and computational work in store.

The effective Reynolds number in homogeneous quantum turbulence at zero temperature

- If dissipation by radiation of sound then must be effective viscosity (or effective Reynolds number)?
- Given energy per unit volume fed from quasi-classical Kolmogorov cascade to the Kelvin-wave cascade: $\varepsilon \sim \dfrac{\kappa^3}{\ell^4}$
- This quantity in classical homogeneous turbulence is $\varepsilon = \nu\langle|\mathrm{curl}\,u|^2\rangle$ ν = kinematic viscosity

For homogeneous quantum turbulence the effective mean square vorticity is $\langle|\mathrm{curl}\,\mathbf{v}_s|^2\rangle = \kappa^2/\ell^4$

Therefore effective kinematic viscosity is $\nu' \sim \kappa \sim 10^{-7}\ \mathrm{m^2 s^{-1}}$

same as the kinematic viscosity of normal ^{4}He

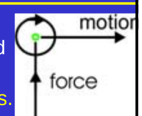
Therefore, sadly, a vanishing conventional viscosity does **not** yield an infinite Re!

Further comments on dissipation in homogeneous quantum turbulence

- Our assumption that dissipation is due entirely to radiation of sound by Kelvin waves is not quite true.
- Two extra sources of dissipation
  - Sound emission *during* reconnections (relatively unimportant in ^{4}He, but very important in trapped atomic gases).
  - In ^{3}He, absorption of Kelvin-wave energy by special (Caroli-Matricon) excitations in the vortex cores.

Finite temperatures-- the normal fluid

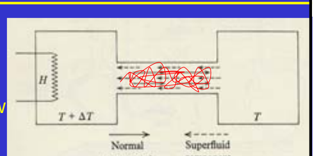
- When a vortex line moves relative to normal fluid it experiences a drag force. Its motion is modified according to the Magnus effect.
  - There is a "mutual friction" between the two fluids.



- If $\mathbf{v}_n = 0$ and $\langle\mathbf{v}_s\rangle = 0$, turbulence is damped. Occurs in ^{3}He (normal fluid has large viscosity). But very different from viscous damping: it acts most strongly on the largest eddies.
- In ^{4}He normal fluid has very small viscosity: both fluids can support turbulence. May have "classical" turbulence in both fluids, on length scales larger than both the vortex spacing and the Kolmogorov dissipation length in the normal fluid. Mutual friction locks the two velocity fields together on large scales, so ^{4}He behaves like a single turbulent fluid. Confirmed by experiment.
- Most interestingly, when $\langle\mathbf{v}_s - \mathbf{v}_n\rangle \neq 0$ the mutual friction can lead to the ***generation*** of turbulence - "counterflow turbulence".

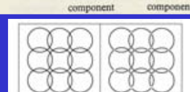
Counterflow turbulence

- Heat in superfluid ^{4}He
- Homogeneous turbulence is ***generated*** by the counterflow



Why?

- Consider an array of identical vortex rings in the presence of normal fluid flowing normal to the plane of the rings.

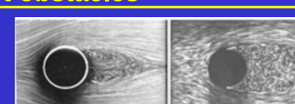


$\otimes V_n$

- If rings have circulation of correct sign, mutual friction will cause them to expand: a mechanism for generating vortex line.
- Reconnections lead to the formation of new rings; hence a self-sustaining turbulence. This is a quite new type of turbulence with no classical analogue.

Flow past obstacles

- This type of turbulence is being pursued



- Satisfactory ways to visualize the turbulence are still in development.
- In ^{3}He rely on drag measurements, although some progress has been made in visualizing the flow at very low temps.
- Most experiments involve oscillating flow, not steady flow. This complicates the interpretation. Need LT linear actuators.
- In some cases the drag looks classical, especially in ^{4}He; in others, especially in ^{3}He, it does not. **Why?**
- Ah, so much to do, and so little funding (Speaking of which, we have funding from EPSRC in UK and NSF in USA. Thanks!)

Summation

- Quantum turbulence involves much new and interesting fluid dynamics, with interesting consequences for turbulent flow.
- On large length scales (containing many quanta) turbulence can appear to be essentially classical.
- But on small length scales it is always dominated by quantum effects. Dissipative processes are associated with these small scales. These processes are usually quite different from classical viscous dissipation, although they can sometimes mimic the effect of classical viscosity.
- Types of turbulence that have no classical analogues can be generated.
- Compared with classical turbulence, the study of quantum turbulence is still in its infancy.

Acknowledgements and references

Experiments on quasi-classical turbulence in ^{4}He: Maurer & Tabeling; Stalp, Skrbek and Donnelly.

Experiments on quasi-classical turbulence in ^{3}He: Lancaster group (Pickett et al)

Interpretation of quasi-classical behaviour: Vinen; Vinen & Niemela; Tsubota et al.

Spin-up turbulence in ^{3}He: Helsinki group (Krusius et al).

Theory of homogeneous turbulence in ^{3}He: Vinen; Volovik et al.

Theoretical discussion of dissipation at very low temperatures: Svistunov; Vinen; Kozik & Svistunov; Nazarenko.

Flow past oscillating obstacles: Schoepe et al; Yano et al; Pickett et al; McClintock et al; Hanninen, Tsubota & Vinen

Turbulence in thermal counterflow: Vinen; Schwarz; Tough.

Vortex reconnections: Koplik & Levine; Barenghi, Adams et al.

For more detailed references see Vinen, *An Introduction to Quantum Turbulence*, J Low Temp Phys: in press.