Kondo Screening in a Magnetically Frustrated Nanostructure: Exact Results on a Stable Non-Fermi-Liquid Phase

Kevin Ingersent,[1] Andreas W. W. Ludwig,[2] and Ian Affleck[3]

[1]Department of Physics, University of Florida, Gainesville, Florida 32611, USA
[2]Department of Physics, University of California, Santa Barbara, California 93106, USA
[3]Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1

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Triangular symmetry stabilizes a novel non-Fermi-liquid phase in the three-impurity Kondo model with frustrating antiferromagnetic interactions between half-integer impurity spins. The phase arises without fine-tuning of couplings, and is stable against magnetic fields and particle-hole symmetry breaking. We find a conformal field theory describing this phase, verify it using the numerical renormalization group, and extract various exact, universal low-energy properties. Signatures predicted in electrical transport may be testable in scanning tunneling microscopy or quantum-dot experiments.

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The same many-body physics that is responsible for the Kondo screening of magnetic impurities in bulk metals [1] produces resonances in tunneling through a quantum dot [2] or an adatom on a metallic surface [3]. Greater experimental control over the latter settings allows systematic study of multiple-"impurity" configurations in which the Kondo effect competes with ordering of the local degrees of freedom [4–6]. One goal that remains elusive is experimental realization of non-Fermi-liquid (NFL) behavior similar to that exhibited, e.g., by the two-impurity and two-channel Kondo models [7,8].

A cluster of three antiferromagnetically coupled spins is of fundamental importance as the simplest example of frustration, a feature of many magnetic systems. Scanning tunneling microscopy (STM) has found two distinct types of compact Cr trimer on a gold surface [5]: "type-2" trimers show a sharp resonance of width (Kondo temperature) $T_K \approx 50$ K, whereas any Kondo effect for isolated Cr atoms and "type-1" trimers seems to have $T_K \ll 7$ K. Attempts to explain this result via variational [9], quantum Monte Carlo [10], and perturbative renormalization-group (RG) [11] treatments of a three-impurity Kondo model have reached opposing conclusions concerning the triangular geometry of the type-2 trimers: equilateral [9,11] or isosceles [10]. Interest is also developing in the interplay between Kondo physics and interdot quantum entanglement in triangular quantum-dot devices [12].

This Letter reports exact results on a frustrated phase of the three-impurity Kondo model exhibiting NFL behavior distinctly different from that found in previously studied models. We present a conformal field theory (CFT), deduced by comparison with numerical renormalization-group (NRG) results, showing that the NFL fixed point is stable against particle-hole-symmetry breaking (unlike its two-impurity counterpart), exchange anisotropy, and even an applied magnetic field (which destroys two-channel Kondo NFL behavior). This enhanced stability compared to other NFL fixed points makes the frustrated phase an excellent candidate for experimental realizations; indeed, it has been argued on the basis of weak-coupling RG [11] to describe the low-energy physics of the type-2 Cr trimers in [5]. We make predictions for the conductance expected in STM experiments on trimers and in certain quantum-dot devices.

Model.—We start with a Hamiltonian $H_{\rm band} + H_{\rm int}$ describing a noninteracting conduction band coupled via

$$H_{\rm int} = J \sum_{j,\alpha,\beta} \psi^{\dagger,\alpha}(\vec{r}_j) \frac{1}{2} \vec{\sigma}_{\alpha\beta} \psi_\beta(\vec{r}_j) \cdot \vec{S}_j \qquad (J > 0) \quad (1)$$

to spin-S impurities $\vec{S}_j$ ($j = 1, 2, 3$) at the vertices $\vec{r}_j$ of an equilateral triangle; $\psi_\alpha(\vec{r})$ annihilates an electron with spin $\alpha = \pm\frac{1}{2}$ at $\vec{r}$. We assume that the permutation group S_3 that maps the set $\{\vec{r}_j\}$ onto itself is a subgroup of the lattice symmetry group (as is the case, e.g., in [5]). The impurities couple to just six orthonormal combinations of conduction states, annihilated by operators $\psi_{h,\alpha} \propto \sum_j e^{-i2\pi jh/3} \psi_\alpha(\vec{r}_j)$, where $h = 0, \pm 1$ is the "helicity": under a $2\pi/3$ rotation about the center of symmetry, a helicity-h state is multiplied by $e^{i2\pi h/3}$. The combined states of the three impurities can also be constructed to have well-defined helicities, in which case the Hamiltonian conserves total helicity (modulo 3) and is invariant under the interchange of all helicity labels 1 and -1. Then, Eq. (1) can be rewritten [13]

$$H_{\rm int} = [J_{00}\vec{s}_{00} + J_{11}(\vec{s}_{11} + \vec{s}_{\bar{1}\bar{1}})] \cdot \vec{S}_0 + [J_{01}(\vec{s}_{01} + \vec{s}_{\bar{1}0}) + J_{1\bar{1}}\vec{s}_{1\bar{1}}] \cdot \vec{S}_1 + [J_{01}(\vec{s}_{10} + \vec{s}_{0\bar{1}}) + J_{1\bar{1}}\vec{s}_{\bar{1}1}] \cdot \vec{S}_{\bar{1}}, \quad (2)$$

where $\vec{S}_h = \sum_j e^{i2\pi jh/3} \vec{S}_j$, $\vec{s}_{hh'} = \sum_{\alpha\beta} \psi^{\dagger,h,\alpha} \frac{1}{2} \vec{\sigma}_{\alpha\beta} \psi_{h',\beta}$, and $\bar{1} \equiv -1$; $J_{hh'}$ equals J times a non-negative factor that depends on the impurity separation and the conduction-band dispersion, as well as h and h' [14].

For $S = \frac{1}{2}$, the NRG shows [13] that over a large region of the parameter space of Eq. (2), the low-energy physics is governed by a "frustrated" fixed point at which the impurities are locked into the subspace of two doublets of combined spin $S_{\rm imp} = \frac{1}{2}$, one each of helicity $h = \pm 1$. Three spins of arbitrary half-integer S, coupled by an additional Hamiltonian term $K \sum_{i<j} \vec{S}_i \cdot \vec{S}_j$ with $K \gg J$, also lock into an $S_{\rm imp} = \frac{1}{2}$, $h = \pm 1$ subspace. Weak-coupling RG analysis [11] of this augmented model, which for $S = \frac{5}{2}$ provides a description of equilateral Cr trimers, is consistent with flow to the same fixed point; for $S = \frac{5}{2}$, moreover, the characteristic temperature $T_{\rm K}$ for this flow is found to greatly exceed the single-impurity Kondo scale, in agreement with the Cr-trimer experiments [5].

In the frustrated phase, $J_{1\bar{1}}$ in Eq. (2) scales to zero, J_{00} and J_{11} can be neglected, and particle-hole asymmetry is marginal [13]. Thus, we analyze the fixed point in a restricted $S_{\rm imp} = \frac{1}{2}$ space, replacing Eq. (2) by

$$H_{\rm int} = -\sqrt{2} J_{01} [(\psi^\dagger \tfrac{1}{2} \vec{\sigma} T^+ \psi) \tau^-_{\rm imp} + {\rm H.c.}] \cdot \vec{S}_{\rm imp}. \quad (3)$$

Here, $T^\pm$ and T^z act on the electron helicity in the spin-1 representation of an "orbital spin" ${\rm SU}^{(t)}(2)$ [15], with matrix elements $(T^z)_{h,h} = h$, $(T^+)_{1,0} = (T^+)_{0,-1} = \sqrt{2}$. The Pauli matrices $\vec{\sigma}$, $2\vec{S}_{\rm imp} \equiv 2\vec{S}_0$, and $\vec{\tau}_{\rm imp}$ act, respectively, on the electron spin, impurity spin, and impurity helicity, with $(\tau^z_{\rm imp})_{h,h} = -h$ for $h = \pm 1$, $(\tau^+_{\rm imp})_{-1,1} = 1$.

It is important to note that setting $J_{1\bar{1}} = 0$ enlarges the S_3 symmetry of Eq. (2) to a ${\rm U}^{(t)}(1)$ symmetry in Eq. (3), replacing total helicity (conserved only modulo 3) by a conserved quantity t_z: the eigenvalue of $\psi^\dagger T^z \psi + \frac{1}{2}\tau^z_{\rm imp}$. Now, $H_{\rm int}$ commutes with ${\rm SU}^{(s)}(2)$ spin, ${\rm U}^{(t)}(1)$ orbital spin, and also with ${\rm SU}^{(i)}(2)$ isospin defined by $I^z = \frac{1}{2}\sum_{h,\alpha} \psi^{\dagger,h,\alpha} \psi_{h,\alpha}$, $I^+ = \frac{1}{2}\sum_{h,\alpha,\beta} \epsilon_{\alpha\beta} \psi^{\dagger,h,\alpha} \psi^{\dagger,-h,\beta}$.

CFT description.—We obtain exact analytical results for the frustrated fixed point using the boundary CFT approach to quantum impurity problems [16]. The operator $\psi_{h,\alpha}$ is considered to act at the boundary $x = 0$ of a one-dimensional space $0 \le x \le l$ [7]. The key is to find a "conformal embedding" [a decomposition of the bulk fermions $\psi_{h,\alpha}(x)$ into products of "constituent" fields [17]] admitting a "fusion procedure" (generating a new conformally invariant boundary condition) that reproduces the fixed-point finite-size spectrum (FSS). We deduce this FSS by extending to higher accuracy the NRG results of [13]. From the fusion procedure, all universal low-energy properties in the physical limit $l \to \infty$ can in principle be computed exactly.

We first construct a conformal embedding of the free Dirac fermions $\psi_{h,\alpha}(x)$ in which the helicities transform in the spin-1 representation of an ${\rm SU}^{(p)}(2)$ "pseudospin" $\vec{P}$, where $P^z = T^z$ and P^+ has matrix elements in the helicity basis $(P^+)_{1,0} = -(T^+)_{1,0}$, $(P^+)_{0,-1} = (T^+)_{0,-1}$. Unlike $\vec{T}$ defined above, $\vec{P}$ commutes with isospin $\vec{I}$. The free-fermion FSS can be decomposed into products of ${\rm SU}^{(s)}(2)_3 \times {\rm SU}^{(i)}(2)_3 \times {\rm SU}^{(p)}(2)_8$ conformal towers [18] as exemplified in Table I for boundary conditions that yield a nondegenerate ground state. Here, ${\rm SU}(2)_k$ is a level-k Kac-Moody CFT; see [17] and references therein.

TABLE I. Finite-size spectrum of free fermions decomposed into products of spin, isospin, and pseudospin conformal towers, labeled by s, i, and p, respectively. The subscript Δ gives each tower's contribution to the excitation energy [19].

$(s)_\Delta$	$(i)_\Delta$	$(p)_\Delta$
$(0)_0$	$(0)_0$	$(0)_0 + (4)_2$
$(\frac{3}{2})_{3/4}$	$(\frac{3}{2})_{3/4}$	$(0)_0 + (4)_2$
$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{3/20}$	$(1)_{1/5} + (3)_{6/5}$
$(1)_{2/5}$	$(1)_{2/5}$	$(1)_{1/5} + (3)_{6/5}$
$(0)_0$	$(1)_{2/5}$	$(2)_{3/5}$
$(1)_{2/5}$	$(0)_0$	$(2)_{3/5}$
$(\frac{1}{2})_{3/20}$	$(\frac{3}{2})_{3/4}$	$(2)_{3/5}$
$(\frac{3}{2})_{3/4}$	$(\frac{1}{2})_{3/20}$	$(2)_{3/5}$

Since Eq. (3) lowers the ${\rm SU}^{(p)}(2)$ symmetry of $H_{\rm band}$ to ${\rm U}^{(p)}(1) \equiv {\rm U}^{(t)}(1)$, we analyze the frustrated fixed point using an embedding obtained from that above by decomposing ${\rm SU}^{(p)}(2)_8 \supset {\rm U}^{(t)}(1)_8 \times Z_8$, where Z_8 is a parafermionic CFT [17]. In any ${\rm SU}(2)_k$ CFT, each primary operator $\phi^{(j)}$, transforming in the spin-j representation ($j = 0, \frac{1}{2}, 1, \ldots, k/2$), factors into a sum of products of a Z_k primary ψ^j_m and a ${\rm U}(1)_k$ boson exponential [20]:

$$\phi^{(j)} = \sum_{j-m \in \mathbb{Z}} \psi^j_m e^{i(m/\sqrt{k})\varphi}. \quad (4)$$

Setting $k = 8$ and $j = p$, we rewrite the spectrum in Table I as products of ${\rm SU}^{(s)}(2)_3 \times {\rm SU}^{(i)}(2)_3 \times {\rm U}^{(t)}(1)_8 \times Z_8$ conformal towers. These products also provide the operator spectrum for the noninteracting model.

Using this conformal embedding, we are able to obtain the frustrated fixed-point FSS from the free-fermion FSS by applying a three-step fusion procedure: (1) fusion with the $s = \frac{3}{2}$ primary operator in ${\rm SU}^{(s)}(2)_3$, then (2) fusion with the $p = \frac{1}{2}$ primary operator in ${\rm SU}^{(p)}(2)_8$, then (3) fusion with the $\psi^{p=0}_{m=2}$ primary operator in Z_8 [21].

As an illustration, Table II lists all CFT states of energy [19] $E < 1$ in the frustrated FSS for boundary conditions that yield a degenerate free-fermion ground state [22], along with energies of NRG levels having the same (s, i, t_z) quantum numbers. To fix the overall NRG energy scale, distorted by band discretization, we match the lowest excitation of the *free-fermion* spectrum to its CFT counterpart. Apart from small energy shifts (residual discretization effects), the CFT and NRG spectra agree perfectly. In fact, all 1810 CFT states with $E \le 1.8$ have been compared with and match NRG levels [23].

Applying the fusion procedure twice to the free-fermion FSS gives the complete and exact spectrum of boundary operators that can be added to the fixed-point Hamiltonian [16]. This spectrum (see Table III) exhibits "fractionalization" of charge, spin, and orbital degrees of freedom, as is

TABLE II. Finite-size spectrum at the frustrated fixed point. $U^{(t)}(1)$ and Z_8 conformal towers are labeled by t_z and (p, m), respectively. Each row represents a pair of states related by a change in the signs of t_z *and* m. E is the CFT excitation energy and $E_{\rm NRG}$ is the NRG energy computed for a band discretization parameter $\Lambda = 3$. See also Table I.

$(s)_\Delta$	$(i)_\Delta$	$(t_z)_\Delta$	$(p, m)_\Delta$	E	$E_{\rm NRG}$
$(0)_0$	$(0)_0$	$(\frac{3}{2})_{9/32}$	$(\frac{1}{2}, -\frac{1}{2})_{7/160}$	0	0
$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{1/32}$	$(\frac{3}{2}, -\frac{3}{2})_{3/32}$	0.1	0.1001
$(0)_0$	$(1)_{2/5}$	$(\frac{1}{2})_{1/32}$	$(\frac{3}{2}, -\frac{3}{2})_{3/32}$	0.2	0.2000
$(1)_{2/5}$	$(0)_0$	$(\frac{1}{2})_{1/32}$	$(\frac{3}{2}, -\frac{3}{2})_{3/32}$	0.2	0.2000
$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{3/20}$	$(\frac{3}{2})_{9/32}$	$(\frac{1}{2}, -\frac{1}{2})_{7/160}$	0.3	0.2996
$(0)_0$	$(0)_0$	$(\frac{1}{2})_{1/32}$	$(\frac{1}{2}, -\frac{3}{2})_{127/160}$	0.5	0.4968
$(0)_0$	$(0)_0$	$(\frac{5}{2})_{25/32}$	$(\frac{1}{2}, +\frac{1}{2})_{7/160}$	0.5	0.5020
$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{1/32}$	$(\frac{3}{2}, +\frac{5}{2})_{19/32}$	0.6	0.5971
$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{3/20}$	$(\frac{3}{2})_{9/32}$	$(\frac{3}{2}, -\frac{1}{2})_{11/32}$	0.6	0.6040
$(1)_{2/5}$	$(1)_{2/5}$	$(\frac{1}{2})_{1/32}$	$(\frac{3}{2}, -\frac{3}{2})_{3/32}$	0.6	0.6001
$(\frac{1}{2})_{3/20}$	$(\frac{3}{2})_{3/4}$	$(\frac{1}{2})_{1/32}$	$(\frac{3}{2}, -\frac{3}{2})_{3/32}$	0.7	0.7004
$(\frac{3}{2})_{3/4}$	$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{1/32}$	$(\frac{3}{2}, -\frac{3}{2})_{3/32}$	0.7	0.7004
$(0)_0$	$(1)_{2/5}$	$(\frac{1}{2})_{1/32}$	$(\frac{3}{2}, +\frac{5}{2})_{19/32}$	0.7	0.7043
$(1)_{2/5}$	$(0)_0$	$(\frac{1}{2})_{1/32}$	$(\frac{3}{2}, +\frac{5}{2})_{19/32}$	0.7	0.7043
$(0)_0$	$(1)_{2/5}$	$(\frac{3}{2})_{1/32}$	$(\frac{3}{2}, -\frac{1}{2})_{11/32}$	0.7	0.6982
$(1)_{2/5}$	$(0)_0$	$(\frac{3}{2})_{1/32}$	$(\frac{3}{2}, -\frac{1}{2})_{11/32}$	0.7	0.6982
$(1)_{2/5}$	$(1)_{2/5}$	$(\frac{3}{2})_{9/32}$	$(\frac{1}{2}, -\frac{1}{2})_{7/160}$	0.8	0.8038
$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{1/32}$	$(\frac{1}{2}, -\frac{3}{2})_{127/160}$	0.8	0.8045
$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{3/20}$	$(\frac{5}{2})_{25/32}$	$(\frac{1}{2}, +\frac{1}{2})_{7/160}$	0.8	0.8116

typical of an NFL fixed point. Remarkably, it also exhibits full $SU^{(p)}(2)$ symmetry. The $SU^{(p)}(2)_8$ operator multiplets (last column of Table III) should again be decomposed using Eq. (4) into $U^{(t)}(1)_8 \times Z_8$.

Boundary operators entering the effective low-energy Hamiltonian for the frustrated fixed point must respect the $SU^{(i)}(2) \times SU^{(s)}(2) \times U^{(t)}(1)$ symmetry of the full Hamiltonian (3). Such operators appear in the first row of Table III. Only $(s, i, t_z, Z_8) = (0, 0, 0, (\psi_0^1)_{1/5})$ is relevant (in the RG sense). It cannot appear because it is odd under the Z_2 subgroup of $S_3 : \psi_{h,\alpha} \rightarrow -\psi_{-h,\alpha}, \tau^-_{\rm imp} \rightarrow \tau^+_{\rm imp}$, which is representable as a π rotation about the x axis in orbital-spin space [23]. The least-irrelevant operator also respecting this discrete Z_2 symmetry of Eq. (3) is the corresponding $SU^{(p)}(2)_8$ descendant of dimension $\Delta = 1 + 1/5$, which yields a correction-to-scaling exponent $1/5$ in excellent agreement with the value 0.200 ± 0.002 observed in the NRG spectrum.

TABLE III. Operator spectrum at the frustrated fixed point. Δ gives each factor's contribution to the scaling dimension. "2×" indicates two operators with the same p and Δ.

$(s)_\Delta$	$(i)_\Delta$	$(p)_\Delta$
$(0)_0$	$(0)_0$	$(0)_0 + (1)_{1/5} + (3)_{6/5} + (4)_2$
$(\frac{3}{2})_{3/4}$	$(\frac{3}{2})_{3/4}$	$(0)_0 + (1)_{1/5} + (3)_{6/5} + (4)_2$
$(\frac{1}{2})_{3/20}$	$(\frac{1}{2})_{3/20}$	$(0)_0 + 2 \times [(1)_{1/5} + (2)_{3/5} + (3)_{6/5}] + (4)_2$
$(1)_{2/5}$	$(1)_{2/5}$	$(0)_0 + 2 \times [(1)_{1/5} + (2)_{3/5} + (3)_{6/5}] + (4)_2$
$(0)_0$	$(1)_{2/5}$	$(1)_{1/5} + 2 \times (2)_{3/5} + (3)_{6/5}$
$(1)_{2/5}$	$(0)_0$	$(1)_{1/5} + 2 \times (2)_{3/5} + (3)_{6/5}$
$(\frac{1}{2})_{3/20}$	$(\frac{3}{2})_{3/4}$	$(1)_{1/5} + 2 \times (2)_{3/5} + (3)_{6/5}$
$(\frac{3}{2})_{3/4}$	$(\frac{1}{2})_{3/20}$	$(1)_{1/5} + 2 \times (2)_{3/5} + (3)_{6/5}$

Physical results.—We now present exact properties that can be deduced from the CFT description. Details, including analysis of the conditions required for observation of these properties, will appear elsewhere [23].

(a) Fixed-point properties.—The frustrated fixed point has an irrational "ground-state degeneracy" [24] $g = [\frac{1}{2} \times (5 + \sqrt{5})]^{1/2}$. Moreover, in a quantum-dot device of triangular symmetry, where biases V_j in leads $j = 1, 2, 3$ produce in lead i a current $I_i = \sum_j G_{ij} V_j$, the $T = 0$ zero-bias conductance is $G_{ii} = 4e^2/3h$. By contrast, the "isospin two-channel" regime [13], in which $J_{1\bar{1}}$ dominates Eq. (2), is unstable against particle-hole asymmetry and at low energy exhibits the Fermi-liquid behavior of the SU(4) fixed point of [12(a)], with $g = 1$ and (in the limit of small particle-hole asymmetry) $G_{ii} = 8e^2/9h < 4e^2/3h$. The other stable fixed point of [13], at which interimpurity correlations are irrelevant and the standard Kondo effect is recovered, has $g = 1$ and $G_{ii} = 0$.

(b) Differential conductance.— The leading irrelevant operator of dimension $\Delta = 1 + 1/5$ governs many properties near the fixed point. In particular, the differential tunneling conductance into the impurities from a metallic lead (e.g., an STM tip located symmetrically with respect to the impurities) in the regime $k_B T, |eV| \ll k_B T_{\rm K}$ (V being the bias voltage) has the form $G_0^{-1} dI/dV \sim 1 - B(T/T_{\rm K})^{1/5} g[AeV/k_B T]$, where G_0 is the $T = 0$ linear-response conductance; A and B are constants that can be fitted to experiment. For $x \rightarrow 0$, $g[x] \rightarrow$ const, so $G_0^{-1} dI/dV \sim 1 - B(T/T_{\rm K})^{1/5} g[0]$, whereas $g[x] \sim cx^{1/5}$ (with c a constant) for $x \rightarrow \infty$, yielding $G_0^{-1} dI/dV \sim 1 - cB(AeV/k_B T_{\rm K})^{1/5}$ [25]. To lowest (quadratic) order in the tunneling matrix element between the impurities and the lead [26], the universal scaling function $g[x]$ equals the exact function given in [16(b)]. Similar (and, in linear response, identical) behavior is expected in transport through triangular quantum-dot devices [23].

(c) Breaking of particle-hole symmetry.—This lowers the isospin SU(2) symmetry to the U(1) subgroup that conserves global charge $2I^z$, while preserving the discrete S_3 symmetry. The spectrum in Table III is reclassified by applying Eq. (4) to $SU^{(i)}(2)_3 \supset U^{(i)}(1) \times Z_3$. The most relevant operators that become allowed in the low-energy Hamiltonian are marginal: the charge current operator $2I^z$, which is exactly marginal and corresponds to a simple

phase shift [16], and a degenerate pair $(s, I^z, Z_3, t_z, Z_8) = ((0)_0, (0)_0, (\psi_0^1)_{2/5}, (0)_0, (\psi_4^2)_{3/5})$ arising from Table III, row 5. The last two operators are the boundary limits of the left- and right-moving bulk currents $J_{L,R} = \psi_{L,R}^\dagger[(T^z)^2 - \frac{2}{3}\mathbf{1}]\psi_{L,R}$ (Table I, row 5) [27]. $J_{L,R}$ generate a U(1) symmetry of the free-fermion bulk theory *not* preserved by the boundary condition. The boundary limits of such operators are exactly marginal [23], consistent with NRG results in the presence of particle-hole asymmetry [13]. Like a phase shift, the three exactly marginal deformations of the boundary conditions affect the FSS (and the boundary limits of $J_{L,R}$ affect the $T = 0$ zero-bias conductance), but not the operator spectrum in Table III [23]. Thus, the NFL fixed point and its signatures, including the ground-state degeneracy and power laws in the conductance, persist away from particle-hole symmetry (unlike, e.g., the NFL behavior of the two-impurity Kondo model [7]).

(d) Other symmetry-breaking perturbations.—It can be deduced from Table III that (i) spin-orbit coupling is relevant with dimension 3/5, (ii) breaking of S_3 symmetry (e.g., through distortion of the equilateral triangular impurity geometry) is relevant with dimension 1/5 [28], (iii) spin-exchange anisotropy is irrelevant, (iv) a Zeeman field acting only on the impurity spins is exactly marginal, and (v) the coupling $J_{1\bar{1}}$ in Eq. (2) is irrelevant. The implications of these results will be discussed elsewhere [23].

In summary, we have found the exact low-energy behavior of a non-Fermi-liquid phase arising from the interplay of magnetic frustration and Kondo physics in the three-impurity Kondo model. The phase is stable against particle-hole asymmetry, exchange anisotropy, and magnetic fields. It should be detectable in tunneling into magnetic adatoms on metallic surfaces and in electrical transport through triangular quantum-dot devices.

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[20] In Eq. (4), m and $m \pm k$ are identified, so m (where $j - m \in \mathbb{Z}$) can be chosen such that $-(k-1)/2 \leq m \leq k/2$. Also, operators ψ_m^j and $\psi_{-(k/2-m)}^{k/2-j}$ are identified.
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[27] See [16] concerning the operator spectra of the free left-moving field ψ_L and its right-moving counterpart ψ_R.
[28] If isosceles triangular symmetry and particle-hole symmetry are preserved, there is just one relevant operator.