SUPPLEMENTARY MATERIAL – Pairing Correlations Near a Kondo-Destruction Quantum Critical Point

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In this supplementary material, we describe in more detail the numerical methods used and present the renormalization-group (RG) flow of the two impurity Bose-Fermi Anderson models.

Methods

Two approaches have been used for our study. The first is an extension of the CT-QMC approach [1–3], for the model with an Ising form of the exchange interaction. After a generalized Lang-Firsov transformation [2, 3], the CT-QMC performs time-dependent perturbation theory in the hybridization and stochastically sums the resulting series via a Monte-Carlo algorithm. In order to locate the $T = 0$ transition via calculations performed at $T \equiv 1/\beta > 0$, we compute the staggered Binder cumulant [3–5] $U_4^s(\beta, I_z, g) = \langle M_s^4 \rangle / \langle M_s^2 \rangle^2$, where the staggered magnetization $M_s = \beta^{-1} \int_0^\beta d\tau\, S_s^z(\tau)$ with $S_s^z = \frac{1}{2}(S_1^z - S_2^z)$. We also calculate the staggered static spin susceptibility $\chi_s(T) = \beta \langle M_s^2 \rangle$. In the presence of a bosonic coupling to S_s, the Heisenberg form of H_{12} is beyond the reach of the CT-QMC.

The second approach is the Bose-Fermi extension [6] of the NRG [7]. The staggered spin susceptibility is calculated as $\chi_s(T) = -\lim_{H_s \to 0} \langle S_s^z \rangle / H_s$ with an additional Hamiltonian term $H_s S_s^z$. The static pairing correlations are obtained by Hilbert transformation of the imaginary part of the dynamical susceptibilities, computed on the real frequency axis in the usual manner [8]. The NRG results presented here were obtained using discretization parameter $\Lambda = 9$, allowing up to 6 bosons per site of the Wilson chain, and keeping up to 1 300 many-body eigenstates after each iteration.

RG Flow

Figure S1 corresponds to the case with Ising interimpurity exchange H_{12}. There are two stable fixed points, indicated by filled circles, which govern the Kondo-screened (Kondo) phase and the local-moment (LM) phase. The Kondo fixed point is at $(g, I_z) = (0, 0)$, while the LM fixed point is at $(g, I_z) = (\infty, 0)$. On the phase boundary, the RG flow is from the Kosterlitz-Thouless (KT) fixed point toward the Kondo-destruction (KD) fixed point. The KT and KD fixed points are both unstable and are shown by open circles. The direction of the RG flow on the phase boundary reflects the discussion in the main text, *i.e.*, all the points for nonzero g on the phase boundary have the same critical behavior as the KD QCP on the g axis.

Figure S2 shows the RG flow for the case with Heisenberg interimpurity exchange H_{12}. In this case, there are three stable fixed points corresponding to the three phases. The Kondo-screened (Kondo), local-moment (LM) and interimpurity-singlet (IS) phases are respectively located at $(g, I) = (0, 0)$, $(g, I) = (\infty, 0)$ and $(g, I) = (0, \infty)$, and are marked by filled circles. On the Kondo-LM phase boundary, the RG flow is from the triple-point (where the three phases meet) to the Kondo-destruction (KD) fixed point. Likewise, the RG flow on the Kondo-IS phase boundary is from the triple-point toward the fixed point KI, which separates the Kondo and IS phases on the I-axis.

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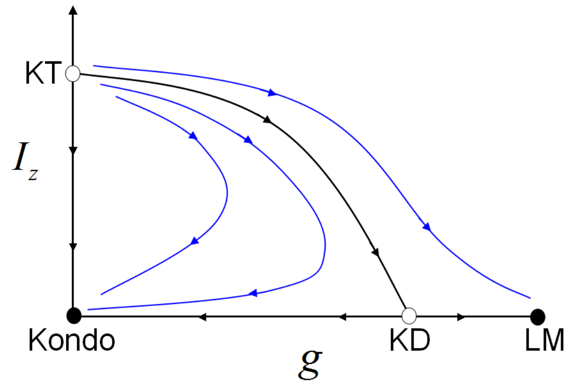

FIG. S1: (color online). Schematic RG flow on the g-I_z plane for the two-impurity Bose-Fermi Anderson model with Ising interimpurity exchange H_{12}. Trajectories with arrows represent the flows of the couplings (g and I_z) with the decrease of energy.

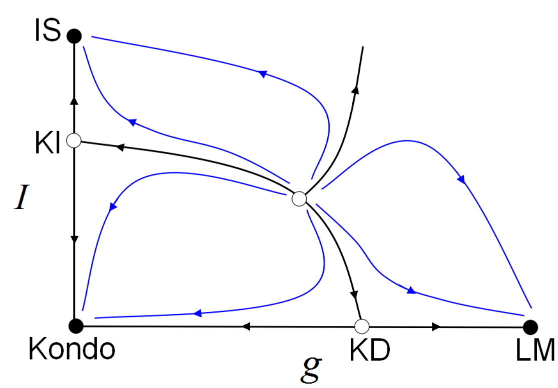

FIG. S2: (color online). Schematic RG flow on the g-I plane for the two-impurity Bose-Fermi Anderson model with Heisenberg inter-impurity exchange H_{12}.