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Renormalization-group study of a magnetic impurity in a Luttinger liquid

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Abstract. – A generalized Anderson model for a magnetic impurity in an interacting one-dimensional electron gas is studied via a mapping onto a classical Coulomb gas. For weak potential scattering, the local-moment parameter regime expands as repulsive bulk interactions become stronger, but the Kondo scale for the quenching of the impurity moment varies non-monotonically. There also exist two regimes dominated by backward potential scattering: one in which the impurity is non-magnetic, and another in which an unquenched local moment survives down to very low temperatures.

The notion of a "Luttinger liquid" has received much attention recently in connection with quantum wires, edge excitations in the fractional quantum Hall effect, and the normal state of high-T_c superconductors [1]. The observation that a single potential scatterer in an interacting one-dimensional (1D) electron system can cause the conductance to vanish [2] has generated particular interest. While the physics of a static impurity is largely understood, that of a dynamical impurity —a magnetic atom being a natural example— remains less clear. Even in a non-interacting host, a magnetic impurity can produce interesting many-body physics, in the form of the Kondo effect [3]. The combined treatment of bulk interactions and impurity-induced correlations, which may be necessary, for instance, to understand the heavy-fermion behaviour observed in $Nd_{2-x}Ce_xCuO_4$ [4], is a considerable theoretical challenge.

Most work on magnetic impurities in 1D systems [5] has focused on the Kondo model, presupposing the existence of a local moment. Repulsive bulk interactions have been found to make possible a Kondo effect for ferromagnetic (as well as antiferromagnetic) exchange couplings J, and to drive the form of the Kondo temperature T_K (below which the exchange renormalizes to strong coupling) from an exponential in J to a power law. Analysis of an Anderson impurity in a restricted Luttinger model suggests that interactions expand the local-moment regime [6]. Certain parameter regimes of the full Anderson model have been

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claimed to renormalize onto an equivalent impurity problem in a Fermi liquid [7], a result that is shown below to be an artefact of neglecting important couplings in the problem.

Here we study a generalized Anderson model, which includes direct and exchange interactions between localized and delocalized electrons in addition to hybridization between the impurity and the Luttinger liquid. We investigate the competition between 1) mixed valence; 2) local-moment formation (and a subsequent Kondo effect); and 3) correlated behaviours arising from strong backward potential scattering [2]. The impurity problem is mapped onto a classical Coulomb gas, which is analyzed using perturbative renormalization-group (RG) methods. We find that repulsive bulk interactions tend to stabilize local-moment formation by suppressing charge transfer between the impurity and the host. With increasing interactions, the Kondo temperature first rises, but then falls due to the reduced hybridization. In addition to a magnetic phase dominated by spin exchange and a non-magnetic phase driven by backward potential scattering, there exists a novel regime which exhibits a Curie susceptibility down to low temperatures, but displays the anomalous transport properties of a static impurity.

We begin with the Hamiltonian $H = H_{\rm Lutt} + H_{\rm imp} + H_{\rm hybrid} + H_{\rm Kondo} + H_{\rm density}$. Here,

$$H_{\rm Lutt} = \sum_{\nu=\rho,\sigma} \frac{v_\nu}{2} \int \left[K_\nu (\nabla\theta_\nu)^2 + K_\nu^{-1} (\nabla\phi_\nu)^2 \right] {\rm d}x \tag{1}$$

is the Luttinger-liquid Hamiltonian for a 1D system, written in terms of four bosonic fields, $\theta_\nu(x)$ and $\phi_\nu(x)$, describing independent charge ($\nu = \rho$) and spin ($\nu = \sigma$) density modes [1]. These fields are related through their gradients to local charge- and spin-density operators for left- and right-moving fermion fields, $\Psi_{-,s}(x)$ and $\Psi_{+,s}(x)$, respectively. (We use $s = \uparrow, \downarrow$ and $+, -$ interchangeably to label the spin-z component.) The Luttinger liquid is characterized by four parameters: the sound velocities v_ν and interaction parameters K_ν for charge and spin modes. In the absence of bulk interactions, $K_\rho = K_\sigma = 1$. Repulsive bulk interactions depress K_ρ into the range $0 < K_\rho < 1$, but $SU(2)$ spin symmetry requires that K_σ still equals unity.

The impurity is described by its energy ε_d, and the on-site repulsion U between d electrons:

$$H_{\rm imp} = \varepsilon_d n_d + \tfrac{1}{2} U n_d (n_d - 1), \quad n_d \in \{0, 1, 2\}\,. \tag{2}$$

The remaining terms in H contain the interaction between the localized orbitals d_s and the fermion fields $\Psi_{p,s}(0)$ at the impurity site ($x = 0$): the hybridization,

$$H_{\rm hybrid} = t\sqrt{a} \sum_{p=\pm} \sum_{s=\uparrow,\downarrow} \left\{ d_s^\dagger \Psi_{p,s}(0) + \Psi_{p,s}^\dagger(0) d_s \right\}; \tag{3}$$

the direct spin-exchange (Kondo) interaction,

$$\begin{aligned} H_{\rm Kondo} = {} & \frac{J_{z\rm F} a}{4} \sum_{p=\pm} \sum_{s,s'} s s' d_{s'}^\dagger d_{s'} \Psi_{p,s}^\dagger(0) \Psi_{p,s}(0) + \frac{J_{\perp\rm F} a}{2} \sum_{p=\pm} \left\{ d_\uparrow^\dagger d_\downarrow \Psi_{p,\downarrow}^\dagger(0) \Psi_{p,\uparrow}(0) + {\rm H.c.} \right\} + \\ & + \frac{J_{z\rm B} a}{4} \sum_{p=\pm} \sum_{s,s'} s s' d_{s'}^\dagger d_{s'} \Psi_{p,s}^\dagger(0) \Psi_{-p,s}(0) + \frac{J_{\perp\rm B} a}{2} \sum_{p=\pm} \left\{ d_\uparrow^\dagger d_\downarrow \Psi_{p,\downarrow}^\dagger(0) \Psi_{-p,\uparrow}(0) + {\rm H.c.} \right\}; \end{aligned} \tag{4}$$

and the density-density interaction,

$$H_{\rm density} = V_{\rm F} a \sum_{p=\pm} \sum_{s=\uparrow,\downarrow} n_d \ :\Psi_{p,s}^\dagger(0) \Psi_{p,s}(0): + V_{\rm B} a \sum_{p=\pm} \sum_{s=\uparrow,\downarrow} n_d \, \Psi_{p,s}^\dagger(0) \Psi_{-p,s}(0)\,. \tag{5}$$

In the above equations, a is a lattice spacing; t is the hybridization matrix element; $J_{z\rm F}$ and $J_{\perp\rm F}$ ($J_{z\rm B}$ and $J_{\perp\rm B}$) are the longitudinal and transverse couplings for forward (backward) exchange

TABLE I. – *Summary of the kink-gas parameters P for the infinite-U case, their bare values $P_{\rm bare}$, and the corresponding* RG *equations. Here y_γ is a fugacity of the kink-gas action; k_γ is the corresponding scaling dimension; E is a symmetry-breaking field; and the coefficients ϵ_σ and ϵ_ρ enter the scaling dimensions, which have the following explicit forms: $k_t = [K_\rho + K_\rho^{-1}(1-2\epsilon_\rho)^2 + 1 + (1-2\epsilon_\sigma)^2]/8$, $k_{\perp\rm F} = [1+(1-2\epsilon_\sigma)^2]/2$, $k_{\perp\rm B} = [K_\rho + (1-2\epsilon_\sigma)^2]/2$, $k_{z\rm B} = k_{\rm V} = (1+K_\rho)/2$, and $k_{\rm P} = 2K_\rho$. Note that k_t, $k_{\perp\rm F}$ and $k_{\perp\rm B}$ renormalize with the bandwidth D, whereas $k_{z\rm B}$, $k_{\rm V}$ and $k_{\rm P}$ do not.*

P	$\mathrm{d}P/\mathrm{d}\ln(D_0/D) =$	$P_{\rm bare}$
y_t	$(1-k_t)y_t + y_t(y_{1\rm V} + \frac{1}{2}y_{z\rm B} + y_{\perp\rm F} + y_{\perp\rm B})e^{-E/2} - y_t y_{0\rm V} e^{E/2}$	$t/\sqrt{2}D_0$
$y_{\perp\rm F}$	$(1-k_{\perp\rm F})y_{\perp\rm F} + 2y_{\perp\rm B}y_{z\rm B} + y_t^2 e^E$	$J_{\perp\rm F}/4D_0$
$y_{\perp\rm B}$	$(1-k_{\perp\rm B})y_{\perp\rm B} + 2y_{\perp\rm F}y_{z\rm B} + y_t^2 e^E + 2y_{\perp\rm B}y_{1\rm P}$	$J_{\perp\rm B}/4D_0$
$y_{z\rm B}$	$(1-k_{z\rm B})y_{z\rm B} + 4y_{\perp\rm F}y_{\perp\rm B} + y_t^2 e^E + 2y_{z\rm B}y_{1\rm P}$	$J_{z\rm B}/4D_0$
$y_{1\rm V}$	$(1-k_{\rm V})y_{1\rm V} + \frac{1}{2}y_t^2 e^E - 2y_{1\rm V}y_{1\rm P}$	$V_{\rm B}/2D_0$
$y_{0\rm V}$	$(1-k_{\rm V})y_{0\rm V} - y_t^2 e^{-E} - 2y_{0\rm V}y_{0\rm P}$	0
$y_{1\rm P}$	$(1-k_{\rm P})y_{1\rm P} - 2y_{1\rm V}^2 + \frac{1}{2}y_{z\rm B}^2 + y_{\perp\rm B}^2$	0
$y_{0\rm P}$	$(1-k_{\rm P})y_{0\rm P} - 2y_{0\rm V}^2$	0
ϵ_σ	$(1-2\epsilon_\sigma)\left(y_t^2 e^E + 2y_{\perp\rm F}^2 + 2y_{\perp\rm B}^2\right)$	$J_{z\rm F}/4D_0$
ϵ_ρ	$(1-2\epsilon_\rho)y_t^2\left(2e^{-E} + e^E\right)$	$K_\rho V_{\rm F}/D_0$
E	$E + 4y_{0\rm V}^2 - 4y_{1\rm V}^2 - y_{z\rm B}^2 - 2y_{\perp\rm F}^2 - 2y_{\perp\rm B}^2$	$\varepsilon_d/D_0 - (J_{z\rm F}/4D_0)^2$
	$+2y_t^2\left(2e^{-E} - e^E\right) + 2y_{0\rm P}^2 - 2y_{1\rm P}^2$	$-K_\rho\left(V_{\rm F}/D_0\right)^2$

scattering; and $V_{\rm F}$ ($V_{\rm B}$) is the forward (backward) potential scattering. Note that $SU(2)$ spin symmetry requires that $J_{z\rm F} = J_{\perp\rm F}$ and $J_{z\rm B} = J_{\perp\rm B}$. The local density $\Psi^\dagger_{p,s}(0)\Psi_{p,s}(0)$ in eq. (5) is normal-ordered with respect to the unperturbed Fermi sea.

The Kondo model studied in refs. [5] is equivalent to $H_{\rm Lutt} + H_{\rm imp} + H_{\rm Kondo}$ with $U = -2\varepsilon_d \to \infty$, while ref. [7] treats $H_{\rm Lutt} + H_{\rm imp} + H_{\rm hybrid}$ with $U = \infty$. We study the full model, eqs. (1)-(5), by extending Si and Kotliar's treatment of an Anderson impurity in a Fermi liquid [8], which combines Haldane's mapping onto a classical Coulomb gas [9] with an RG procedure due to Cardy [10]. Below we sketch the main steps of our calculation, which is non-perturbative in the bulk interactions. A detailed account will be given elsewhere.

We separate the Hamiltonian into an unperturbed part H_0, plus a perturbation H_1 which contains all terms that change the number of electrons in any of the four conduction-electron branches ($+,\uparrow$; $+,\downarrow$; $-,\uparrow$; and $-,\downarrow$). Thus, H_0 comprises $H_{\rm Lutt}$, the $J_{z\rm F}$ Kondo terms, and the $V_{\rm F}$ potential-scattering terms. We expand the partition function Z in powers of H_1, and express it as a sum over all possible histories. A history is a sequence of "kinks" —each kink corresponding to one application of a term in H_1— which overall preserves the impurity configuration and the occupation of each conduction branch. The response of the host to local distortions gives rise to long-range interactions between the kinks within a given history. Hence, just as in a Fermi liquid [8], Z takes the form of a fugacity expansion for the partition function of a 1D multicomponent gas of kinks with logarithmic interactions. The Luttinger-liquid case is substantially more complicated, though, due to i) the existence of both left- and right-moving fermions, which forces one to follow the history of the branch occupation numbers as well as that of the impurity; ii) the greater number of fugacities (one for each type of kink); and iii) the explicit K_ρ-dependence of the coupling between kinks.

The endpoint of these manipulations is a set of differential equations describing the renormalization of dimensionless kink-gas parameters with increasing short-time cut-off, the inverse of the bandwidth $D = \pi v_\nu/a$. (We assume equal spin and charge velocities.) Table I lists

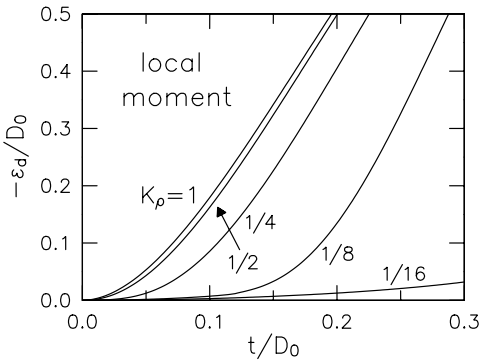

Fig. 1

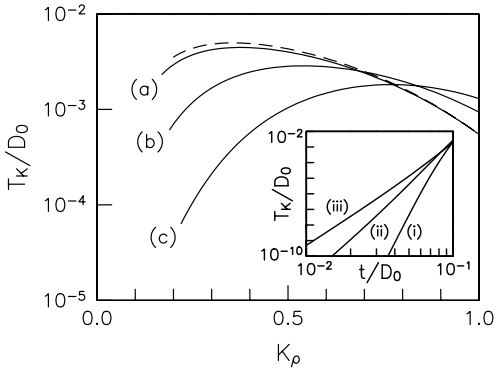

Fig. 2

Fig. 1. – Boundary of the local-moment regime, for $U = \infty$, zero bare exchange and potential scattering, and different bulk interactions K_ρ. The local-moment regime lies above each curve.

Fig. 2. – Kondo temperature $T_{\rm K}$ *vs.* bulk interactions K_ρ, for zero bare exchange and potential scattering. Solid curves: $U = \infty$ and (a) $\varepsilon_d = -0.5D_0$, $t = 0.15D_0$; (b) $\varepsilon_d = -0.24D_0$, $t = 0.10D_0$; (c) $\varepsilon_d = -0.06D_0$, $t = 0.05D_0$. Broken curve: $U/2 = -\varepsilon_d = 0.5D_0$, $t = 0.11D_0$. Inset: log-log plot of $T_{\rm K}$ *vs.* hybridization t, for $U = \infty$, $\varepsilon_d = -0.24D_0$, and (i) $K_\rho = 3/4$; (ii) $K_\rho = 1/2$; (iii) $K_\rho = 1/3$.

these equations, restricted for brevity to the case $U = \infty$. The effect of bulk interactions is to change certain of the scaling dimensions k_γ, each of which determines the relevance or irrelevance of a fugacity y_γ. The first five fugacities in table I represent terms in the original Hamiltonian. Additional terms are generated upon renormalization, among which the most relevant are backward potential scattering from an empty impurity ($y_{0\rm V}$) and singlet $4k_{\rm F}$ pair-scattering from an empty or singly occupied impurity ($y_{0\rm P}$ or $y_{1\rm P}$). (The pair-scattering processes represent Hamiltonian terms proportional to $\Psi^\dagger_{+,\uparrow}(0)\Psi^\dagger_{+,\downarrow}(0)\Psi_{-,\downarrow}(0)\Psi_{-,\uparrow}(0) + \text{H.c.}$)

Table I contains three other parameters: ϵ_σ and ϵ_ρ enter the effective charges associated with the kinks. They correspond, respectively, to $J_{z\rm F}$ Kondo scattering and the forward-potential-scattering difference between singly occupied and empty impurity states. A symmetry-breaking field E represents the energy difference between the singly occupied and empty states.

By construction, the kink-gas approach treats $J_{z\rm F}$ and $V_{\rm F}$ on a different footing from their backward-scattering and spin-flip counterparts, leading to non-systematic third-order (in y and ϵ) terms in the RG equations. On truncation to second order, though, our equations preserve $SU(2)$ spin symmetry, unlike those derived by Lee and Toner for the Kondo model [5].

Note that the bulk interaction parameter K_ρ, and hence the scaling dimensions $k_{z\rm B}$, $k_{\rm V}$, and $k_{\rm P}$, do not renormalize. Thus, in contrast to Li's equations [7] (which rescale y_t, E, and a quantity related to k_t, but neglect other couplings that are necessarily generated), the full set of RG equations can never renormalize a Luttinger-liquid host onto a Fermi liquid.

Given bare values of the kink-gas parameters, the coupled RG equations in table I can be iterated as the bandwidth D is reduced from its bare value D_0. The low-temperature physics is determined by which parameter first becomes of order unity (signaling the breakdown of the fugacity expansion). For instance, a local moment forms when at least one parameter from among $-E$, $|y_{\perp\rm F}|$, $|y_{\perp\rm B}|$, $|y_{z\rm B}|$, $|y_{1\rm V}|$ and $|y_{1\rm P}|$ grows to $+1$, while all the others remain smaller in magnitude. As shown in fig. 1, the range of parameters t and ε_d satisfying this criterion grows monotonically as K_ρ decreases. This expansion of the local-moment regime —initially slight, but more dramatic for $K_\rho \lesssim 0.25$— stems from a reduction in k_t, the scaling dimension of y_t. The precise value of k_t (see table I) derives from the power law density of states of

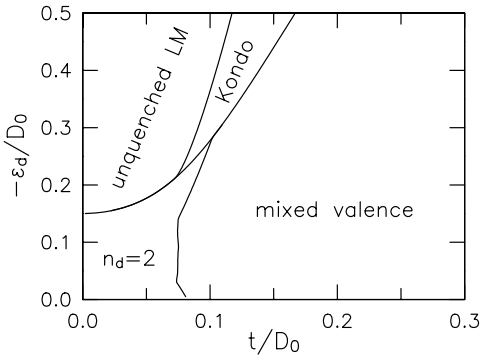

Fig. 3. – Phase diagram, ε_d *vs.* t, for $K_\rho = 1/3$ and a symmetric Anderson impurity ($U = -2\varepsilon_d$) with $V_{\rm F} = V_{\rm B} = 0.1D_0$. See the text for an explanation of the different phases.

a Luttinger liquid [1]. Physically, however, this trend with K_ρ simply reflects the increased difficulty of transferring an electron between the impurity and a correlated host. Similar effects have been found in a restricted Luttinger model [6] and in a Fermi liquid with a power law density of states [11]. We expect them to occur in other interacting systems, as well.

In a Fermi liquid, starting from weak coupling, the only alternatives to local-moment formation are mixed valence (y_t becomes large) and an empty impurity (E grows to $+1$). A Luttinger liquid can support additional phases in which more than one RG parameter is large (*e.g.*, y_t and $y_{0\rm V}$). Generalized mixed-valence behaviour occurs even for bulk interactions so strong ($K_\rho < 3 - \sqrt{8} \approx 0.17$) that y_t is formally irrelevant ($k_t > 1$).

If a local moment does form, around some energy scale $T_{\rm LM}$, the possibility arises of a Kondo effect for $D \lesssim T_{\rm K} < T_{\rm LM}$. Within the Kondo model [5], $T_{\rm K}$ increases monotonically with bulk interactions and, when $J \ll (1 - K_\rho)D_0$, crosses over from an exponential in J (the form found in a Fermi liquid) to a power law: $T_{\rm K} \approx D_0[J/(1-K_\rho)D_0]^{2/(1-K_\rho)}$. Since our RG equations extend to the local-moment regime (once the empty impurity state is eliminated via a Schrieffer-Wolff transformation), we can study the dependence of $T_{\rm K}$ on the parameters of the more fundamental Anderson model.

Figure 2 plots the variation of $T_{\rm K}$ with K_ρ for fixed ε_d and t. The non-monotonicity seen both for $U = \infty$ and for the opposite limit of a symmetric impurity, $U = -2\varepsilon_d$, reflects two competing effects of bulk repulsion: reduced hybridization at high energies, $D \gtrsim T_{\rm LM}$, which depresses the effective exchange $J_{\rm LM}$ on entry to the local-moment regime; and accelerated renormalization of the Kondo couplings for $D \lesssim T_{\rm LM}$ due to enhanced $2k_{\rm F}$ spin correlations. For weak interactions (K_ρ close to 1), the latter effect wins, and the behaviour of $T_{\rm K}$ is similar to that found in the Kondo model [5]. (Note that $T_{\rm K}$ can rise an order of magnitude above its non-interacting value, in contrast to the 10% effect obtained in ref. [7] by assuming renormalization onto a Fermi-liquid host.) For stronger interactions, the depression of $J_{\rm LM}$ dominates: $T_{\rm K}$ is still approximately proportional to $t^{4/(1-K_\rho)}$ (see inset in fig. 2) —consistent with the pure-Kondo case [5]— but non-universal prefactors reverse the overall trend with K_ρ. The position of the maximum in $T_{\rm K}$ depends on the relative ranges of D over which the system remains in the high-energy and local-moment regimes. For $K_\rho \lesssim 0.2$, the local-moment regime can still be entered, but subsequently the singlet pair-scattering $y_{1\rm P}$ becomes comparable to $y_{z\rm B}$ and $y_{\perp\rm B}$, making it unclear whether or not a Kondo effect takes place.

To this point we have considered only cases with $V_{\rm F} = V_{\rm B} = 0$. In a Fermi liquid, potential scattering is exactly marginal and can generally be absorbed into a redefinition of the bare impurity parameters. In a Luttinger liquid, by contrast, backward potential scattering is

relevant [2], and has the same scaling dimension as backward Kondo scattering. There is thus a delicate balance between magnetic and non-magnetic scattering.

Figure 3 shows the ε_d *vs.* t phase diagram for a symmetric impurity ($U = -2\varepsilon_d$) with $V_{\rm F} = V_{\rm B} = 0.1D_0$. The boundaries mark fairly rapid crossovers between different regimes. In addition to the Kondo and generalized mixed-valence phases described above, there are two regimes dominated by potential scattering. The first occurs when $|\varepsilon_d|$ and t are sufficiently small that the backward potential scattering is able to freeze the impurity in a doubly occupied ($n_d = 2$) configuration. The problem then becomes equivalent to strong static potential scattering, *i.e.* the electrical conductance should vanish with temperature T as $T^{1/K_\rho - 1}$ [2]. This regime, in which the impurity is non-magnetic, grows upward and to the right in fig. 3 as $V_{\rm B}$ increases or K_ρ decreases. The second regime cannot possibly develop in a Fermi liquid, starting from weak coupling. A local moment is formed, but subsequently the backward potential scattering from the singly occupied impurity ($y_{1\rm V}$ and $y_{1\rm P}$) becomes strong before the Kondo coupling can quench the local moment. Over an extended temperature range the conductance should again vary as $T^{1/K_\rho - 1}$, but in this regime the susceptibility should be Curie-like. (At very low energies, other screening processes may eventually quench the impurity moment [12].) As the bulk interactions become stronger, this unquenched region occupies an increasing fraction of the local-moment regime.

In summary, we have studied the competition between magnetic and non-magnetic scattering from an impurity atom placed in an interacting 1D electron gas. Bulk correlations hinder charge transfer to and from the impurity, tending to stabilize the local-moment regime against mixed-valence behaviour. However, strong backward potential scattering disfavours a local moment. In an interesting intermediate phase, a local moment forms but the subsequent growth of backward potential scattering overwhelms the Kondo effect, leaving an unquenched moment down to very low temperatures. When a Kondo effect does take place, the characteristic scale $T_{\rm K}$ varies non-monotonically with the bulk interaction strength.

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REFERENCES

[1] For a recent review of Luttinger liquids, see Voit J., *Rep. Prog. Phys.*, **58** (1995) 977.
[2] Kane C. L. and Fisher M. P. A., *Phys. Rev. Lett.*, **68** (1992) 1220; *Phys. Rev. B*, **46** (1992) 15233; Furusaki A. and Nagaosa N., *Phys. Rev. B*, **47** (1993) 4631; Milliken F. P., Umbach C. P. and Webb R. A., *Solid State Commun.*, **97** (1996) 309.
[3] Kondo J., *Prog. Theor. Phys.*, **32** (1964) 37.
[4] Brugger T. *et al.*, *Phys. Rev. Lett.*, **71** (1993) 2481; Fulde P., Zevin V. and Zwicknagl G., *Z. Phys. B*, **92** (1993) 133.
[5] Lee D.-H. and Toner J., *Phys. Rev. Lett.*, **69** (1992) 3378; Furusaki A. and Nagaosa N., *Phys. Rev. Lett.*, **72** (1994) 892; Schiller A. and Ingersent K., *Phys. Rev. B*, **51** (1995) 4676; Fröjdh P. and Johannesson H., *Phys. Rev. Lett.*, **75** (1995) 300.
[6] Phillips P. and Sandler N., *Phys. Rev. B*, **53** (1996) 468.
[7] Li Y. M., *Phys. Rev. B*, **52** (1995) 6979.
[8] Si Q. and Kotliar G., *Phys. Rev. Lett.*, **70** (1993) 3143; *Phys. Rev. B*, **48** (1993) 13881.
[9] Haldane F. D. M., *J. Phys. C*, **11** (1978) 5015.
[10] Cardy J. L., *J. Phys. A*, **14** (1981) 1407.
[11] Gonzalez-Buxton C. and Ingersent K., *Phys. Rev. B*, **54** (1996) 15614.
[12] Fabrizio M. and Gogolin A., *Phys. Rev. B*, **51** (1995) 17827.