PHY 2060 Fall 2006 — Final Exam

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all ten questions, each of which carries a maximum of 10 points. Write your solution below each question, continuing on additional paper if necessary. Please try to write neatly!

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem, g (the acceleration due to gravity near the Earth's surface), G (the universal gravitational constant), and/or c (the speed of light). For numerical problems, take $g = 10\,\mathrm{m/s^2}$, $G = 6.7 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$, and $c = 3.0 \times 10^8\,\mathrm{m/s}$.

During this exam, you may use up to four formula sheets and an electronic calculator. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any other electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

1. The Savuka gold mine in South Africa extends 3.8 km below the Earth's surface. Find the fractional difference $\Delta g/g$ between the acceleration due to gravity at the mine's bottom and that at the surface. Assume for the purposes of this question that the Earth is a uniform sphere of radius 6,400 km.

2. The engine of a 1500-kg car delivers 30 kW of useful power to its drive wheels. Find the maximum road speed at which the car can climb a 5° incline. Neglect air resistance.

3. A meter rule has a 0.6-kg mass hanging from the 90-cm mark and a 1.1-kg mass hanging from the 10-cm mark. You find that the rule balances in a horizontal orientation when supported at the 40-cm mark. What is the mass of the meter rule?

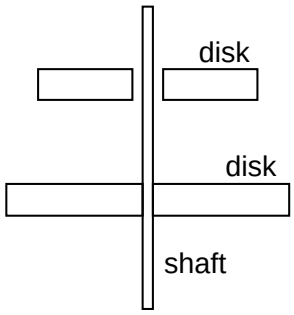
4. A printing company attempts to increase profitability by speeding up its printing presses. One press is printing "Go Gators" labels that will later be stuck onto badges. Paper rolls at speed $0.8c$ past a circular stamp of diameter 2 inches, which descends instantaneously, prints a label, and then rises instantaneously to be ready to print the next label. However, when the press is stopped, the labels have to be thrown out because in their own rest frame they are oval-shaped, not circular.

   (a) What is the dimension of each label in the direction at right angles to its motion during printing?

   (b) What is the dimension of each label in the direction parallel to its motion during printing?

   Give both your answers in the rest frame of the labels.

5. The lower disk in the figure has mass 190 g and radius 2.5 cm. It is initially rotating at 150 rev/min on a light, frictionless shaft of negligible radius. The upper disk, of mass 110 g and radius 2.0 cm, is at rest. It is allowed to drop freely down the shaft onto the lower disk, and frictional forces act to bring the two disks to a common angular velocity. The disks are of uniform thickness and density.



   (a) What is the final angular velocity of the two disks?

   (b) What fraction of the initial kinetic energy is lost to friction?

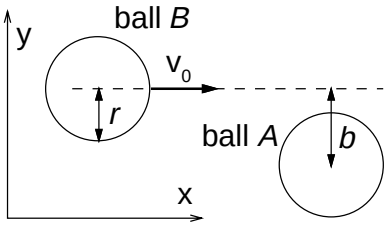
6. Two identical stars, each of mass M, rotate about their mutual center of mass in a common circular orbit of radius R. At all times, the stars are at diametrically opposite positions in the orbit.

   (a) Find the period of each star's orbit.

   (b) Find the total mechanical energy of this system (taking the gravitational potential energy to be zero at infinite separation).

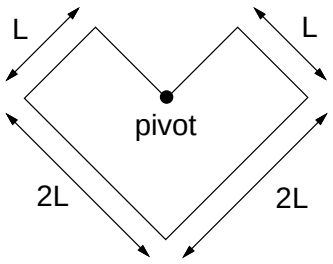
7. Jim throws a tennis ball at 55° to the horizontal so that it lands 32 m away on level ground. If Jim throws the tennis ball with the same initial speed and direction, how far away will it land on a 20° upward slope? Neglect air resistance.

8. Ball A of radius r is at rest on a horizontal, frictionless surface when it is struck by an identical ball B traveling at speed v_0 parallel to the x axis (see diagram). The *impact parameter*—the shortest distance between the balls' centers if they were to pass through one another without colliding—is b. Find the final velocity (not just the speed) of each ball, assuming that the collision is elastic and the balls slide without rotating.

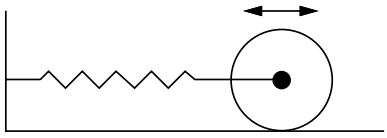


   Hint: Consider the direction of the forces between the two balls. Also, you may find it helpful to recall that $\tan^2\theta + 1 = \sec^2\theta = 1/\cos^2\theta$.

9. A thin, uniform plate is cut and suspended from a frictionless pivot as shown in the figure. Find the frequency of small oscillations about the plate's equilibrium position.



10. A solid cylinder of mass M and radius R is mounted on a frictionless axle passing through its center so that the cylinder can roll on a level tabletop. The axle, of mass m and negligible radius, is attached to one end of a horizontal, ideal spring of spring constant k. The other end of the spring is attached to a rigid support. Suppose that the axle is displaced horizontally a small distance from its equilibrium position, and is then released. In its subsequent motion, the cylinder rolls without slipping over the table.



(a) Write down the kinetic energy of this device as a function of the axle's velocity v.

(b) Use energy conservation to deduce the period of oscillation of the axle.