PHY 2060 Fall 2006 - Exam 2

1. A muon of mass 1.88×10^{-28} kg enters the Earth's atmosphere from outer space with a momentum 1.39×10^{-19} kg·m/s. What is the muon's speed in m/s?

Using the non-relativistic definition of linear momentum

$$p = mv$$

we find

$$v = \frac{p}{m} = \frac{1.39 \times 10^{-19} \text{ kg m/s}}{1.88 \times 10^{-28} \text{ kg}}$$

$$= 7.4 \times 10^{8} \text{ m/s}$$

$$> \text{speed of light}$$

$\Rightarrow$ We must use the relativistic definition

$$p = \gamma_v mv = \frac{mv}{\sqrt{1-(v/c)^2}}$$

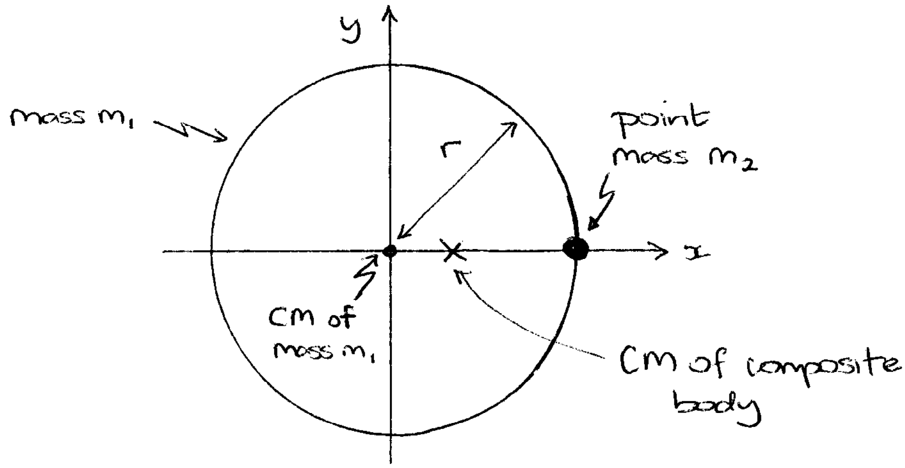
$$\left[1-\left(\frac{v}{c}\right)^2\right]\left(\frac{p}{c}\right)^2 = \left(\frac{mv}{c}\right)^2$$

$$v = \frac{p}{\sqrt{(p/c)^2 + m^2}}$$

$$= \frac{1.39 \times 10^{-19} \text{ kg m/s}}{\sqrt{\left[(1.39 \times 10^{-19} \text{ kg m/s})/(3.0 \times 10^{8} \text{ m/s})\right]^2 + (1.88 \times 10^{-28} \text{ kg})^2}}$$

$$= 2.78 \times 10^{8} \text{ m/s}$$

2. A soccer ball has a circumference of 69 cm and a mass of 430 g. As a prank, it has been doctored by having a 100-gram point mass glued to one location on the interior surface. How far from the ball's center is the center of mass of the combined object? Treat the undoctored ball as a perfect sphere.



The cm position of a body composed of two sub-parts is

$$\vec{r}_{cm} = \frac{1}{m_1+m_2}\left(m_1\vec{r}_{cm,1} + m_2\vec{r}_{cm,2}\right)$$

With the doctored ball oriented as shown above

$$x_{cm,1} = 0$$

$$x_{cm,2} = r$$

$$\Rightarrow \quad x_{cm} = \frac{m_2 r}{m_1+m_2}$$

and by symmetry

$$y_{cm,1} = y_{cm,2} = y_{cm} = 0$$

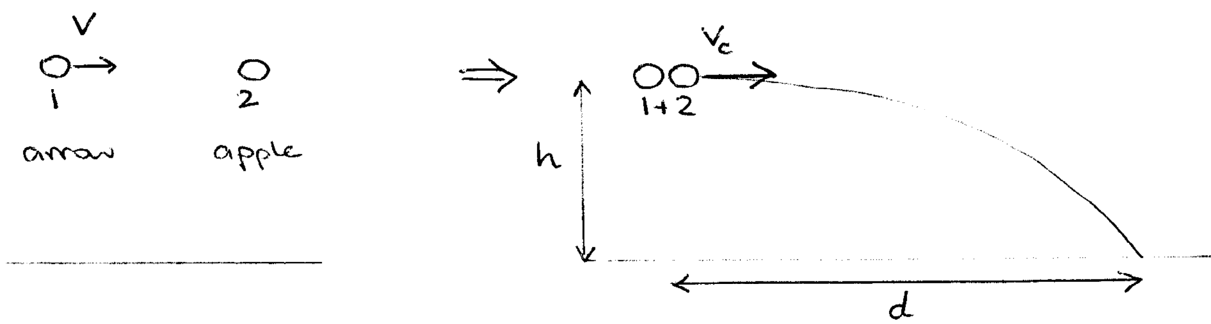
Thus, the distance from the ball's center to the combined cm is

$$x_{cm} = \frac{m_2 r}{m_1+m_2}$$

$$= \frac{100g}{430g+100g} \quad \frac{69\,cm}{2\pi}$$

$$= 2.1\ cm$$

3. At an archery exhibition, an archer fires a 25-gram arrow that buries itself in a 100-gram apple resting on a horizontal, frictionless platform situated 1.25 m above the (level) ground. At the moment of impact, the arrow is traveling at 75 m/s in a direction parallel to the ground. How far horizontally from the apple's initial position do the apple and the arrow hit the ground?



The arrow and the apple undergo a totally inelastic collision. By conservation of momentum, the common velocity of both objects right after the collision is directed horizontally with velocity

$$V_c = \frac{m_1 V_i}{m_1 + m_2}$$

The subsequent motion is free-fall under gravity. The time until the objects hit the ground is given by

$$y = y_0 + V_{0y} t - \tfrac{1}{2} g t^2$$

$$0 = h + 0 - \tfrac{1}{2} g t^2$$

$$t = \sqrt{\frac{2h}{g}}$$

The x-component of the velocity is constant, so the horizontal distance traveled is

$$d = V_c t$$

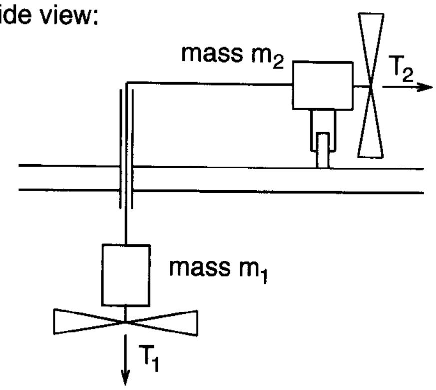
$$= \frac{m_1 V_i}{m_1 + m_2} \sqrt{\frac{2h}{g}}$$

$$= \frac{(25g)(75 m/s)}{25g + 100g} \sqrt{\frac{2(1.25m)}{10 m/s^2}}$$

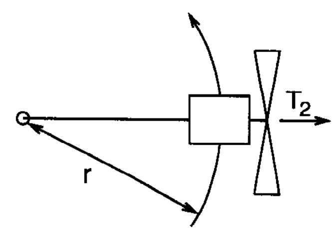
$$= 7.5 m$$

4. Two model-aircraft engines are connected by an ideal string that runs through a hole in a horizontal, frictionless tabletop. Engine 1 (of mass m_1) hangs from one end of the string while its propeller exerts a downwards thrust T_1. Engine 2 (of mass m_2) rotates on the tabletop at a constant speed v around the circumference of a circle of radius r while its propeller exerts a thrust T_2 radially outward from the center of motion. Find r in terms of the other quantities specified.

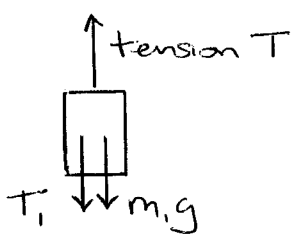
side view:



view from above:



Engine 1:

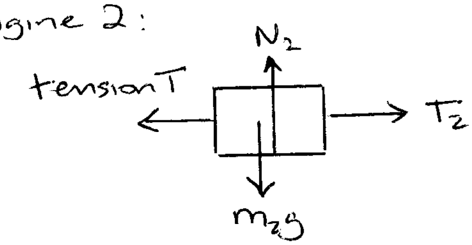


N2 in the vertical direction:

$$T - T_1 - m_1 g = m_1 a_1 = 0$$

$$T = T_1 + m_1 g \quad (1)$$

Engine 2:



N2 in the radial direction:

$$T_2 - T = -\frac{m_2 v^2}{r}$$

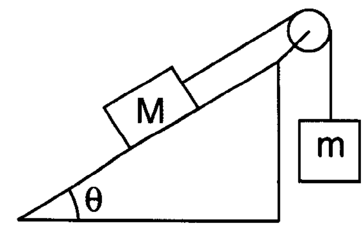
centripetal force

$$r = \frac{m_2 v^2}{T - T_2} \quad (2)$$

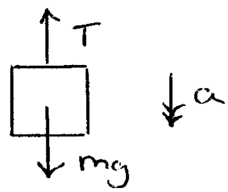
Combining (1) and (2),

$$r = \frac{m_2 v^2}{m_1 g + T_1 - T_2}$$

5. Two blocks, connected by an ideal string running over a massless, frictionless pulley, are released from rest in the initial positions shown in the figure. What is the subsequent acceleration of the mass M for the case where $\theta = 30°$, $M = 2.0\,\text{kg}$, $m = 0.4\,\text{kg}$, the coefficient of static friction between mass M and the slope is 0.3, and the corresponding coefficient of kinetic friction is 0.2?

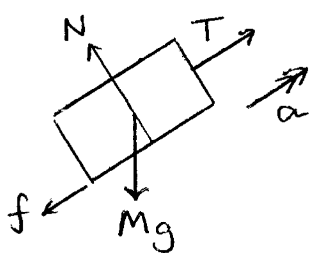


Since the string is ideal, the blocks have the same acceleration a and experience the same tension T.



N2 vertically:

$$mg - T = ma \qquad (1)$$



N2 perpendicular to the slope:

$$N - Mg\sin\theta = 0 \qquad (2)$$

N2 parallel to the slope:

$$T - Mg\sin\theta - f = Ma \qquad (3)$$

We must first test whether static equilibrium holds, i.e., $a = 0$.

(1) $\Rightarrow$ $\quad T = mg$

(3) $\Rightarrow$ Required friction $\quad f_s = mg - Mg\sin\theta$

$$= [0.4\,\text{kg} - (2.0\,\text{kg})\sin 30°](10\,\text{m/s}^2)$$

$$= -6.0\,\text{N}$$

The maximum magnitude of the static frictional force is

$$f_{max} = \mu_s N = \mu_s Mg\cos\theta \qquad \text{by (2)}$$

$$= (0.3)(2.0\,\text{kg})(10\,\text{m/s}^2)\cos 30°$$

$$= 5.2\,\text{N}$$

Since the required friction $|f_s| > f_{max}$, static equilibrium <u>does not</u> hold. The frictional force becomes

$$f = -\mu_k N = -\mu_k Mg\cos\theta$$

where the minus sign reflects the fact that M is going to slide down the slope, so f will point upwards.

Then (1) + (2) $\Rightarrow$

$$a = -\frac{M(\sin\theta - \mu_k\cos\theta) - m}{M + m}\,g$$

$$\approx -1.1\,\text{m/s}^2$$

6. At a candy factory, chocolate-covered nuts, each of mass 10 g, are dropped at a rate of 30 nuts/second into a cardboard box resting on a scale, which has been adjusted to read zero when the box is empty. The nuts are dropped from rest out of the chocolate-coating machine at a height of 50 cm above the box, and land on the bottom of the box without rebounding. What will be the reading on the scale (in newtons) right after the fiftieth nut has landed? The scale's response is sufficiently slow that you can treat the flow of matter as continuous rather than being made up of discrete nuts.

After the nuts have fallen a height h under gravity, starting from rest, their speed is

$$v = \sqrt{2gh}$$

The reading on the scale equals the normal force exerted by the scale on the box minus the weight of the box (since the scale was zeroed when the box was empty).

normal force = weight of stationary nuts in box + rate of change of momentum of nuts on contact with box

$$= n m g + \frac{dn}{dt} m v$$

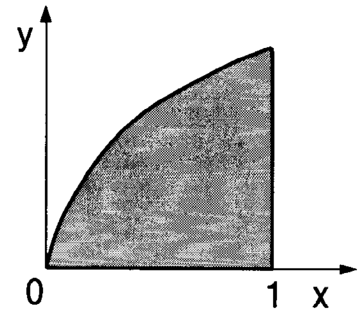
where m = mass of one nut and $n(t)$ is the number of nuts in the box at time t.

$$\Rightarrow \quad \text{scale reading} = m\left(ng + \frac{dn}{dt}\sqrt{2gh}\right)$$

$$= (0.010\,\text{kg})\left[(50)(10\,\text{m/s}^2) + (30\,\text{s}^{-1})\sqrt{2(10\,\text{m/s}^2)(0.50\,\text{m})}\right]$$

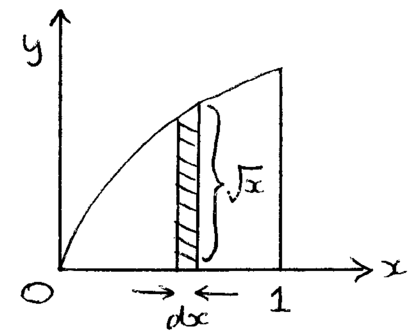
$$= 5.95\ \text{N}$$

7. A plate of uniform thickness is bounded by the lines $y = 0$, $y = \sqrt{x}$, and $x = 1$, as shown in the figure. Find the x and y coordinates of the plate's center of mass.



Since the plate is uniform,

$$\vec{r}_{cm} = \frac{\int \vec{r}\, dA}{\int dA} \qquad dA = \text{area element}$$



$$\begin{aligned} A &= \int dA \\ &= \int_0^1 \sqrt{x}\, dx \\ &= \left[\tfrac{2}{3} x^{3/2}\right]_0^1 \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} x_{cm} &= \frac{1}{A}\int_0^1 x\sqrt{x}\, dx \\ &= \frac{3}{2}\left[\tfrac{2}{5} x^{5/2}\right]_0^1 \\ &= \frac{3}{5} \end{aligned}$$

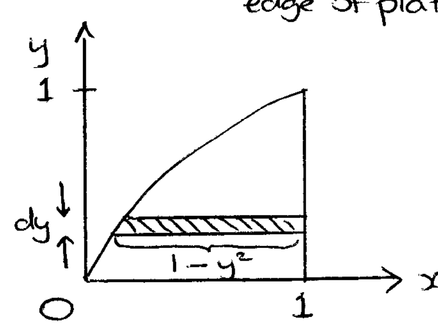
$y = \sqrt{x}$

$\Downarrow$

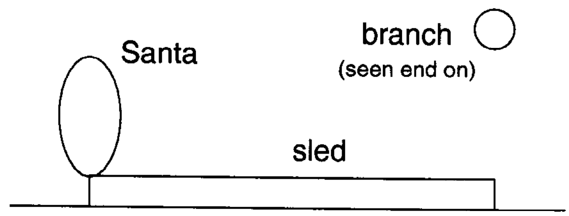
$x = y^2$

along curving edge of plate

$$\begin{aligned} y_{cm} &= \frac{1}{A}\int_0^1 y(1-y^2)\, dy \\ &= \frac{3}{2}\left[\tfrac{1}{2}y^2 - \tfrac{1}{4}y^4\right]_0^1 \\ &= \frac{3}{8} \end{aligned}$$



8. Santa and his sled are stranded on the surface of a frozen pond. (Santa's reindeer are nowhere in sight.) The sled is 4 m long, and (fully loaded with toys) has a mass of 1000 kg. Santa (whose mass is 100 kg) is standing at the rear of the sled.



A tree branch hangs right over the front end of the sled. Afraid that the ice will crack, Santa scrambles forward to try to grab onto the branch. Assuming that there is negligible friction between the ice and the sled, how far horizontally from the tree branch will Santa find himself by the time he reaches the front end of the sled?

Santa and the sled form a system that experences no net horizontal force.

$$\Rightarrow \qquad x_{cm} = \text{constant}$$

Let the branch be at $x = 0$. Then the CM position, based on the initial situation shown above, is

$$x_{cm} = \frac{m_{santa}\, x_{cm,santa} + m_{sled}\, x_{cm,sled}}{m_{santa} + m_{sled}} \qquad (1)$$

$$= \frac{m_{santa}(-L) + m_{sled}(-L/2)}{m_{santa} + m_{sled}}$$

Let the final position of Santa and the front end of the sled (measured relative to the branch) be x_f. Then

$$x_{cm} = \frac{m_{santa}\, x_f + m_{sled}(x_f - L/2)}{m_{santa} + m_{sled}} \qquad (2)$$

Equating x_{cm} in (1) and (2),

$$(m_{santa} + m_{sled})\, x_f - m_{sled}\frac{L}{2} = -(m_{santa} + \tfrac{1}{2} m_{sled})\, L$$

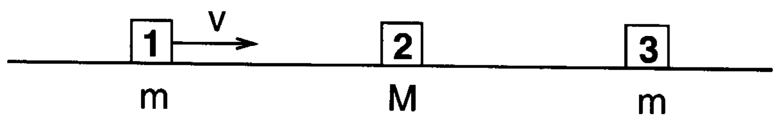
$$x_{cm} = \frac{-m_{santa}\, L}{m_{santa} + m_{sled}}$$

$$= \frac{-100\,\text{kg}}{100\,\text{kg} + 1000\,\text{kg}}\, 4.0\,\text{m}$$

$$= -0.36\,\text{m}$$

So Santa is still 36 cm from the branch.

9. Three carts are spaced out as shown in the figure along a straight track that permits the carts to move (without friction) in only one dimension. Carts 1 and 3 have mass m, and cart 2 has mass $M > m$. Carts 2 and 3 are initially stationary, while cart 1 is initially moving towards cart 2 at speed v. Assume that all subsequent collisions are elastic. What value of M/m ensures that carts 1 and 3 have the same final speed?



First 1 and 2 collide elastically. Using the standard formulae

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} + \frac{2m_2}{m_1 + m_2} V_{2i}$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i} + \frac{m_2 - m_1}{m_1 + m_2} V_{2i},$$

the velocities after the first collision are

$$V_1^{(1)} = \frac{m - M}{M + m} V \quad < 0 \quad \text{since } M > m$$

$$V_2^{(1)} = \frac{2m}{M + m} V$$

$$V_3^{(1)} = 0.$$

Then 2 and 3 collide, after which

$$V_1^{(2)} = -\frac{M - m}{M + m} V$$

$$V_2^{(2)} = \frac{M - m}{M + m} V_2^{(1)} = \frac{2M(M-m)}{(M+m)^2} V$$

$$V_3^{(2)} = \frac{2M}{M + m} V_2^{(1)} = \frac{4mM}{(M+m)^2} V$$

We want

$$|V_1^{(2)}| = |V_3^{(2)}|$$

$$\frac{M - m}{M + m} = \frac{4mM}{(M+m)^2}$$

$$(M-m)(M+m) = 4mM$$

$$M^2 - 4mM - m^2 = 0$$

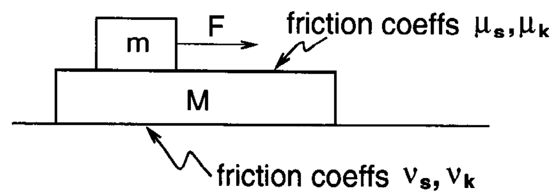
$$\frac{M}{m} = 2 \pm \sqrt{2^2 + 1} \quad \leftarrow \text{choose '+' root so that } M > m$$

$$\frac{M}{m} = 2 + \sqrt{5} \simeq 4.2$$

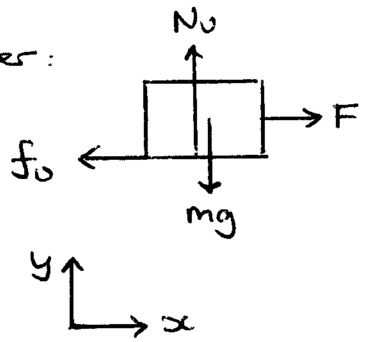
10. A block of mass m lies on a block of mass M, which in turn sits on a horizontal table, as shown in the figure. The coefficients of static and kinetic friction between the two blocks are μ_s and μ_k, respectively, with $\mu_s > \mu_k$. The coefficients of static and kinetic friction between the lower block and the table are ν_s and ν_k, respectively, with $\nu_s > \nu_k$. A string attached to the upper block exerts a horizontal force on the block. The magnitude F of this force is chosen so as to give the lower block its maximum possible acceleration.

(a) What is the magnitude of the maximum possible acceleration that the lower block can experience?

(b) What value of F is required to give the lower block its maximum acceleration?



upper:

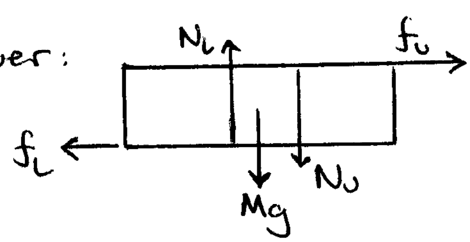


N2....

along x: $\quad F - f_U = m a_U \quad$ ①

along y: $\quad N_U - mg = 0 \quad$ ②

lower:



N2....

along x: $\quad f_U - f_L = M a_L \quad$ ③

along y: $\quad N_L - N_U - Mg = 0 \quad$ ④

Since the lower block is sliding on the table,

$$f_L = \nu_k N_L = \nu_k (M+m) g$$

from ② and ④

(a) ③ ⇒ We maximize the acceleration a_L by maximizing f_U, which happens at the limit of static friction:

$$f_U^{max} = \mu_s N_U = \mu_s mg,$$

in which case

$$a_L^{max} = a_U = \frac{1}{M}\left[\mu_s m - \nu_k (M+m)\right] g$$

(b) ① ⇒ $\quad F = m a_U + f_U$

At the limit of static friction, $a_U = a_L^{max}$ and $f_U = f_U^{max}$:

$$F = (\mu_s - \nu_k)\left(1 + \frac{m}{M}\right) mg$$