## PHY 2060 Spring 2006 — Exam 2

Instructions: Attempt all ten questions, each of which carries a maximum of 10 points. You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem and/or $g$, the acceleration due to gravity near the Earth's surface. For numerical problems, take $g=10 \mathrm{~m} / \mathrm{s}^{2}$. Please try to write neatly!

During this exam, you may use two formula sheets and an electronic calculator. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any other electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: On my honor, I have neither given nor received unauthorized aid in doing this assignment.

1. A plate, of uniform thickness and uniform density, is in the shape of a $6 \times 6$ square with a $2 \times 1$ hole, as shown in the diagram. Find the center of mass position relative to the bottom left corner of the plate.

2. A space station consists of four "life pods" (each of which is essentially a spherical shell of mass $m$ and radius $r$ ) spaced at equal distances around a "space bridge" (a narrow circular tube of mass $M$ and radius $R$ ). Calculate the approximate rotational inertia of the space station about an axis (the dashed line in the diagram) that passes through the center of mass of the station and also through the centers of two of the life pods.

3. A plate, of uniform thickness and uniform density, has a shape in the $x y$ plane defined by the lines $x=0, y=3$, and $y=\frac{1}{3} x^{2}$. Find the $x$ coordinate of the plate's center of mass.
4. Dick, of mass 40 kg , and Jane, of mass 35 kg , stand at opposite ends of a $4.0-\mathrm{m}$-long cart of mass 50 kg . The cart can move without friction along rails that lie along the (horizontal) $x$ axis. Initially, Dick is located at coordinate $x=0$. He then throws a $5.0-\mathrm{kg}$ pumpkin at $5.0 \mathrm{~m} / \mathrm{s}$ along the $+x$ direction to Jane. You may ignore gravity in this problem.
(a) What is the velocity of the cart the instant after Dick throws it?
(b) How long is the pumpkin in the air before Jane catches it?
(c) What is the velocity of the cart the instant after Jane catches it?
(d) What is Dick's final location?
5. Ball 1 of mass 3 kg traveling at $2 \mathrm{~m} / \mathrm{s}$ in the $+x$ direction, approaches ball 2 of mass 1.5 kg traveling at $1 \mathrm{~m} / \mathrm{s}$ in the opposite direction. The balls undergo a head-on collision (so all motion is confined to one dimension). The collision is not explosive.
(a) What is the smallest speed that ball 2 could possibly have after the collision?
(b) What is the largest speed that ball 2 could possibly have after the collision?
6. A table consists of a uniform tabletop of mass $M$, measuring $L \times L$, and supported by four thin legs, each of mass $M / 8$ and located halfway from the center of the tabletop to a corner (as shown from above in the diagram). What is the largest mass $m$ that can be placed halfway along one edge of the tabletop if the table is to remain stable?

7. Two uniform, frictionless balls made of the same material rest as shown in the diagram in the bottom of a rigid container. The radius of the lower ball is twice that of the upper ball. Find the forces exerted on the spheres by (a) the container bottom, (b) the container sides, and (c) each other. Express each answer in terms of the weight $W$ of the smaller ball.

8. A uniform meter-stick of mass 250 g has a small hole drilled through it at the $25-\mathrm{cm}$ mark. A fixed pin passing through the hole acts as a frictionless pivot for rotation of the meter-stick in a vertical plane. A $500-\mathrm{g}$ point mass is attached to the meter-stick at the 10cm mark and a $75-\mathrm{g}$ point mass is attached at the $75-\mathrm{cm}$ mark. The meter-stick is initially oriented so that it makes an angle of $30^{\circ}$
 to the horizontal, with the $500-\mathrm{g}$ mass lower than the $75-\mathrm{g}$ mass (see diagram). It is then released from rest. Find the initial angular acceleration of the system in $\mathrm{rad} / \mathrm{s}^{2}$. Which of the two masses (the $500-\mathrm{g}$ or the $75-\mathrm{g}$ ) initially rises higher than its starting point?
9. A solid cylinder of mass $M$ and radius $R$ rotates about its symmetry axis. A mass $2 M$ is hung from an ideal string wrapped around the outside of the cylinder (see figure). This system is released from rest. How long does it take for the mass to descend a distance $d$ ?

10. A uniform sphere of mass $M$ and radius $R$ rolls without slipping down a plane inclined at angle $\theta$ to the horizontal. Find the minimum value of the coefficient of static friction between the sphere and the surface that allows this motion to take place.
