## DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

**Instructions:** Attempt all ten questions, each of which carries a maximum of 10 points. Write your solution below each question, continuing on additional paper if necessary. Please try to write neatly!

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. (This may benefit you even in the case of the multiple-choice Question 1.) Give all final algebraic answers in terms of variables defined in the problem, g (the acceleration due to gravity near the Earth's surface), and/or c (the speed of light). For numerical problems, take  $g = 10 \text{ m/s}^2$  and  $c = 3.0 \times 10^8 \text{ m/s}$ .

1. The figure shows a box of mass  $m_b$  on a frictionless ramp inclined at an angle  $\theta$  to the horizontal. An ideal rope runs from the box, around a fixed pulley and a suspended pulley, to a fixed attachment point on the ceiling. A pail of mass  $m_p$  hangs by an ideal string from the suspended pulley. The pulleys are massless and frictionless.

Place a check to the left of any/all of the following statements that **must** be true:

- i. The tension in the string is  $m_p g$ .
- ii. The tension in the string is twice that in the rope.
- iii. The acceleration of the pail is half as great (in magnitude) as the acceleration of the box.
- iv. The tension in the rope is  $m_b g \cos \theta$ .
- v. The acceleration of the box must be smaller in magnitude than g (the free-fall acceleration due to gravity).
- 2. A stream of darts, each of mass 75 g and traveling horizontally at 60 m/s, strike a door at a rate of 6 darts per second. Half the darts bury themselves in a dartboard mounted on the door, while the other half strike metal and recoil elastically, traveling back in the direction they came from. Find the average horizontal force experienced by the door due to the impact of the darts.



- 3. The figure shows a block composed of copper and gold slabs joined together. The density of copper is  $8.96 \,\mathrm{g/cm^3}$ , and that of gold is  $19.32 \,\mathrm{g/cm^3}$ . Find the Cartesian coordinates of the block's center of mass. Make sure you specify the origin of your coordinate system.
- 4. A point mass m is suspended from a horizontal rod by two ideal strings (as shown in the figure): a diagonal string that makes an angle  $\theta$  to the rod, and is attached to the rod at the end marked A in the figure; and a vertical string that is attached to the rod a distance dfrom end A. Find the tension in the vertical string when the rod is rotated about the end Ain such a way that the mass moves in a horizontal circle at a constant speed v.
- 5. Block A (mass 5.0 kg) rests on a horizontal table top. The coefficient of kinetic friction between A and the table is 0.35. Block B (mass 3.5 kg) is connected to A via an ideal string running over a massless, frictionless pulley, as shown in the figure. Block C (mass 1.25 kg), when placed on A, is just heavy enough to prevent the system from moving.



- (a) Determine the coefficient of static friction between block A and the table.
- (b) If block C is removed, what is the subsequent acceleration of block A?
- 6. You wish to pull a box of mass M up an inclined plane at a constant speed by pulling with a force F on an ideal rope attached to the box. The plane makes an angle  $\theta$  to the horizontal, and the coefficient of kinetic friction between the box and the plane is  $\mu_k$ . You are free to choose the angle  $\phi$  between the rope and the plane, and hence the angle  $\theta + \phi$  between the rope and the horizontal (see the figure).



- (a) Find an expression for F as a function of  $\phi$ .
- (b) What value of  $\phi$  minimizes the force F that you must apply to raise the box at constant speed?

7. Three carts are spaced out as shown in the figure along a straight track that permits the carts to move (without friction) in only one dimension. Carts 1, 2, and 3 have masses 2m, m, and 2m, respectively. Carts 1 and 3 are initially stationary, while cart 2 is initially moving towards cart 3 at speed v, as shown in the figure. Find the final velocity of each cart, assuming that all collisions are elastic.



8. A pion  $(\pi^+)$ , with momentum  $2.5 \times 10^{-31}$  kg m/s, decays by emitting a positron  $(e^+)$  and an electron neutrino  $(\nu_e)$  at angles of 58° and 20°, respectively, to the pion's initial direction of motion (see figure). What is each decay particle's velocity? For the purposes of this problem, assume that the mass of a positron is 0.511 MeV  $(9.15 \times 10^{-31}$  kg) and the mass of an electron neutrino is 10.0 eV $(1.79 \times 10^{-35}$  kg). Ignore relativistic effects.



- 9. Two ice pucks A and B, of identical mass, are fired towards one another, each sliding at speed  $v_1$  across horizontal, frictionless ice. They collide, and afterwards the two pucks travel off in opposite directions, each moving at speed  $2v_1/3$ . In a later experiment, puck B is stationary on the ice when puck A is launched towards it at speed  $v_2$  along the positive x axis. After a collision, the motion of both pucks is confined to the x axis. Find the final velocities (magnitudes and directions along the x axis) of the two pucks.
- 10. A two-dimensional object is formed by the region of the x-y plane that (i) is enclosed by the curves  $y = 4x - x^3$  and  $y = x^3 - 4x$ , and (ii) satisfies  $x \ge 0$ . Find the x coordinate of this object's center of mass.