# PHY 2060 FALL 2007 - EXAM 1

1. A solid sphere, released from rest just below the surface of the water in a bottomless well, begins to fall vertically downwards. The well water is completely still, apart from the disturbance created by the sphere. The only forces acting on the sphere are its weight, buoyancy, and drag. Assume that the acceleration due to gravity does not change with depth in the well.

   Place a check to the left of any/all of the following statements that is/are true:

   ✗ i. The sphere will eventually come to a halt.

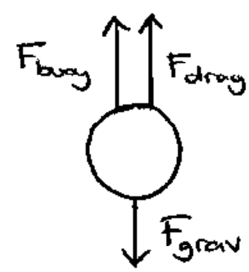
   ✓ ii. The sphere's acceleration is greatest in magnitude the moment after it is released.

   ✗ iii. The sphere will undergo a constant acceleration.

   ✗ iv. If the sphere is of sufficiently low density, it may eventually halt its downward motion and then make its way back up to the surface.

   ✓ v. The sphere will eventually approach a constant, nonzero velocity.

$F_{grav}$ and $F_{buoy}$ do not change during the fall, but $F_{drag}$ increases as the speed increases. Therefore, the downward acceleration becomes ever smaller in magnitude and eventually approaches zero.




i) There is never a net force upward on the sphere so it cannot slow to a halt. <u>FALSE</u>

ii) Initially, the sphere is stationary so the is no drag. The acceleration is greatest at this moment. <u>TRUE</u>

iii) Since the drag force changes with speed, the acceleration is not constant. <u>FALSE</u>

iv) See i) above. <u>FALSE</u>

v) When the acceleration approaches zero, the sphere reaches its terminal speed. <u>TRUE</u>

2. During a 2030 World Cup match, Robo-Pelé kicks a soccer ball at a speed of 225 km/h directly towards his opponents' goal, which is 25 m away. How long does it take for the ball to reach the goal?

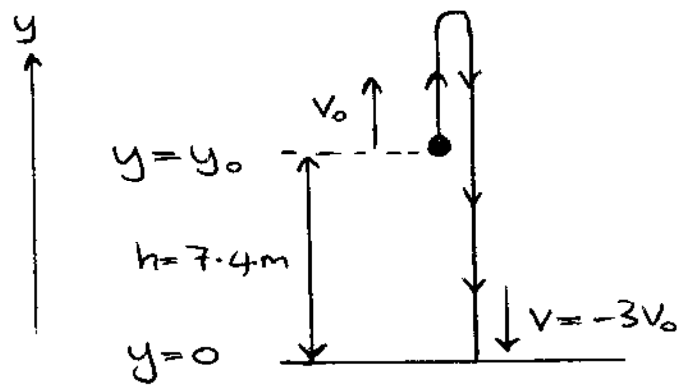
Since there is no information to the contrary, we must assume the ball has a constant velocity.

Then $\quad \text{time} = \dfrac{\text{displacement}}{\text{speed}}$

$$= \frac{25\,\text{m}}{225\,\text{km/h}} \times \frac{3600\,\text{s}}{1\,\text{h}} \times \frac{1\,\text{km}}{1000\,\text{m}}$$

$$= 0{\cdot}40\,\text{s}$$

3. A boy leaning over the edge of a balcony throws a ball vertically upwards. By the time the ball hits the ground 7.4 m below, it is moving at three times its initial speed. Find that initial speed.




For this free-fall problem, we can use

$$V^2 = V_o^2 - 2g(y - y_o)$$

$$(-3V_o)^2 = V_o^2 + 2gh$$

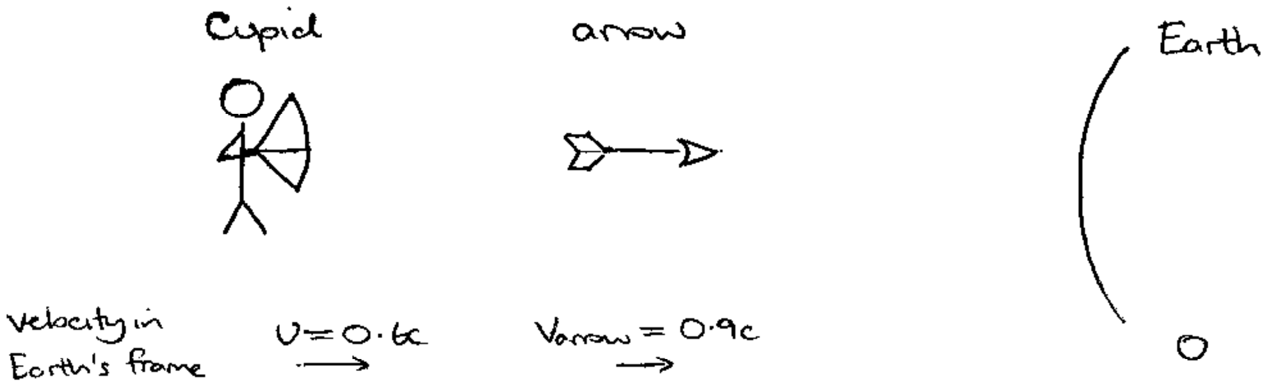
$$8V_o^2 = 2gh$$

$$V_o = \frac{1}{2}\sqrt{gh}$$

$$= \frac{1}{2}\sqrt{(10 m/s^2)(7.4 m)}$$

$$= 4.3 m/s$$

4. A tracking station on Earth spots Cupid cruising directly towards Earth at a speed of $0.6c$. The tracking station also detects an arrow that Cupid has fired towards a work-obsessed physicist. The tracking station measures the arrow traveling at a speed of $0.9c$ straight towards Earth. How fast does Cupid observe the arrow to be traveling relative to himself?




Cupid's rest frame moves to the right at $v = 0.6c$ relative to Earth's rest frame.

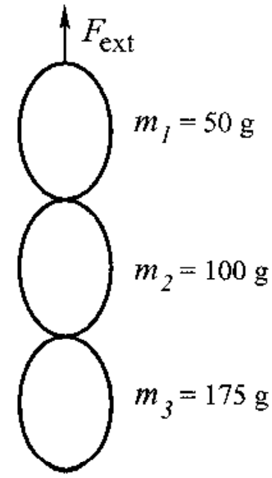
Using the Lorentz velocity transformation, the arrow's velocity in Cupid's rest frame is

$$v'_{arrow} = \frac{v_{arrow} - v}{1 - v v_{arrow}/c^2}$$

$$= \frac{0.9c - 0.6c}{1 - (0.6)(0.9)}$$

$$= 0.65c \text{ or } 2.0 \times 10^8 \text{ m/s}$$

5. A chain consisting of three links (numbered 1, 2, 3) is lifted vertically with a constant acceleration of 5.6 m/s$^2$, as shown in the figure.




(a) Find the magnitude of the external force acting on the top link ($F_{ext}$ in the diagram).

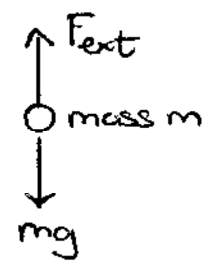
(b) Find the magnitude of the net force acting on each of the links (call them $F_{net,1}$, $F_{net,2}$, $F_{net,3}$).

(c) Find the magnitude of the two interlink forces (call them $F_{12}$ and $F_{23}$).

(a) To find the force Fext exerted by an external agent to cause the acceleration, we can treat the three links as a single object of mass

$$m = m_1 + m_2 + m_3 = 325\text{g} = 0.325\text{ kg}$$

N2 applied to this object gives

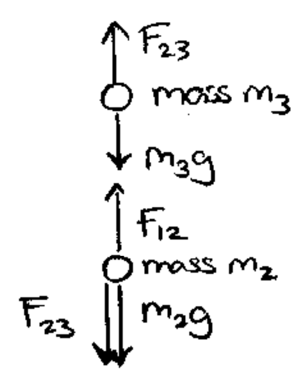



$$F_{ext} - mg = ma$$
$$F_{ext} = m(a+g)$$
$$= (0.325\text{kg})(5.6+10)\text{ m/s}^2$$
$$= 5.1\text{ N}$$

(b) By N2, link j (= 1, 2 or 3) experiences a net force

$$F_{net,j} = m_j a$$
$$F_{net,1} = (0.050\text{kg})(5.6\text{m/s}^2) = 0.28\text{N}$$
$$F_{net,2} = (0.100\text{ kg})(5.6\text{m/s}^2) = 0.56\text{ N}$$
$$F_{net,3} = (0.175\text{ kg})(5.6\text{m/s}^2) = 0.98\text{ N}$$

(c) Applying N2 to link 3,




$$F_{23} - m_3 g = m_3 a$$
$$F_{23} = m_3(a+g) = 2.7\text{N}$$

Applying N2 to link 2

$$F_{12} - F_{23} - m_2 g = m_2 a$$
$$F_{12} = F_{23} + m_2(a+g) = (m_2+m_3)(a+g) = 4.3\text{ N}$$

6. A conveyor belt has the letters of the alphabet printed in order along its length in a repeating sequence, one letter every 3 cm. The belt moves at a constant speed of 30 cm/s in a direction so that the letters pass a fixed reference point in the order $ABCD\ldots$ (as opposed to $AZYX\ldots$). A machine drops a can from rest at a height of 80 cm above the belt. If the letter $C$ is directly below the can at the moment of release, what letter will the can land on?

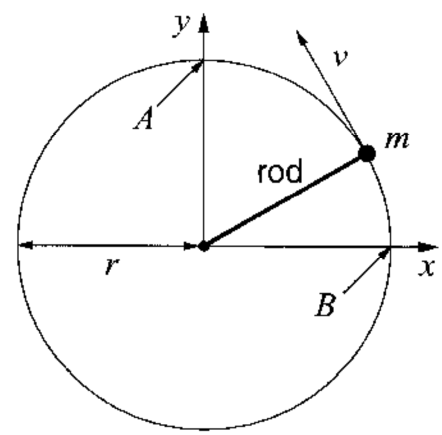
From
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2,$$
the time to fall a distance $y_0 - y = h$ from rest ($v_{0y} = 0$) is
$$t = \sqrt{\frac{2h}{g}}.$$

In the time the can falls a distance $h$, the belt moves a distance
$$d = v_b t \qquad v_b = \text{speed of belt}$$

so the number of letters that pass by is
$$n = \frac{d}{s} \qquad s = \text{letter separation}$$
$$= \frac{v_b}{s}\sqrt{\frac{2h}{g}}$$
$$= \frac{30\,\text{cm/s}}{3\,\text{cm}}\sqrt{\frac{2 \times 80\,\text{cm}}{1000\,\text{cm/s}^2}}$$
$$= 4$$

$\Rightarrow$ The letter under the can when it lands will be G.

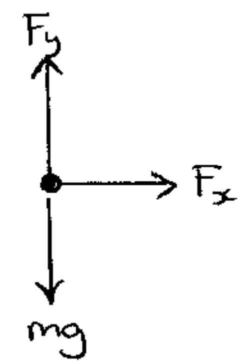
7. A point mass $m$ is attached to one end of a massless, rigid rod. The other end of the rod is attached to a motor mounted on a benchtop. When the motor is switched on, the mass moves in a vertical plane, rotating at constant speed $v$ in a circle of radius $r$. Let $F_x$ and $F_y$ be the horizontal and vertical components (respectively) of the force exerted by the rod on the mass. (See the coordinate axes in the figure.)




(a) Find $F_x$ and $F_y$ at the moment when the mass is at the point labeled $A$ in the figure.

(b) Find $F_x$ and $F_y$ at the moment when the mass is at the point labeled $B$ in the figure.

To maintain uniform circular motion, the net force on the mass must be $\frac{mv^2}{r}$ towards the center of the circle. The forces acting are that due to the rod and gravity.




(a) At point A, $\quad F_x = 0$

and $\quad F_y - mg = -\frac{mv^2}{r}$

$\Rightarrow \quad F_y = m\left(g - \frac{v^2}{r}\right)$

(b) At point B, $\quad F_x = -\frac{mv^2}{r}$

and $\quad F_y - mg = 0$

$\Rightarrow \quad F_y = mg$

8. Proxima Centauri is 4.2 light years from Earth, as measured in the Earth's rest frame. (A light year is the distance light travels in a vacuum in one calendar year.) At the instant a rocket heading straight for Earth passes Proxima Centauri, the rocket's on-board computer makes an announcement that crew member Alice is exactly 20 years old. How fast must the rocket travel in the Earth's rest frame if the rocket is to make it back to Earth just as the on-board computer announces Alice's twenty-first birthday?

Let $L$ be the Earth–P.C. distance measured on Earth

$\tau'$ be the journey time measured on the rocket

$v$ be the velocity of the rocket along the Earth's $x$ axis.

Method I: Length contraction. In the rocket's rest frame, the Earth-P.C. distance is $L' = L/\gamma_v$

Earth is approaching the rocket at speed $v$, so the journey time is

$$\tau' = L'/v = L/\gamma_v v \qquad (1)$$

Method II: Time dilation. In the Earth's frame the journey time is

$$\tau = \gamma_v \tau'$$

But $\tau = \frac{L}{v}$, so

$$\gamma_v \tau' = \frac{L}{v} \qquad (2)$$

Method III: Lorentz transformation. Let the events be (1) rocket passes P.C. and (2) rocket reaches Earth.

In the Earth's frame $\quad \Delta x = L, \quad \Delta t = ?$

In the rocket's frame $\quad \Delta x' = 0, \quad \Delta t' = \tau'$

Note: rocket doesn't move in its own rest frame

Now

$$\Delta x = \gamma(\Delta x' + v\,\Delta t')$$

$$L = \gamma_v v \tau' \qquad (3)$$

(1), (2), (3) all give

$$v\tau' = \sqrt{1-(v/c)^2}\, L$$

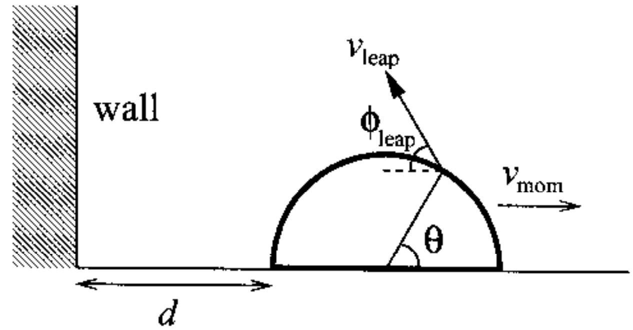
Solving for $v$, find

$$v = \frac{c}{\sqrt{1+(c\tau'/L)^2}}$$

Since $L = 4.2$ light years and $\tau' = 1$ year, $L/c\tau' = 4.2$

$$\Rightarrow \qquad v = 0.973\,c \text{ or } 2.9 \times 10^8\,\text{m/s}$$

9. A mother spider carries her baby on her back. The mother, who had been resting quietly a distance $d = 3\,\text{cm}$ from a wall, notices someone approaching with a broom to squash her. She starts running away from the wall with a velocity $v_{\text{mom}} = 1.63\ \text{cm/s}$.




The baby, realizing that doom is imminent, decides to make a break for it by jumping onto the wall. If the baby jumps 1.0 second after his mother starts running, what is the minimum leap speed $v_{\text{leap}}$ relative to the ground that will allow him to land on the wall? Treat the mother spider as a hemisphere of radius 2.0 cm. The baby is located on the mother's surface at angle $\theta = 49°$ and leaps towards the wall at an initial angle of $\phi_{\text{leap}} = 49°$ (see figure).

Choose the coordinate origin to be at the foot of the wall. Since the mother runs for $T = 1\,\text{s}$ before the baby jumps, the baby's launch coordinates are

$$x_0 = d + V_{\text{mom}} T + r_{\text{mom}}(1 + \cos\theta) = 7.94\,\text{cm}$$
$$y_0 = r_{\text{mom}} \sin\theta = 1.51\,\text{cm}$$

The baby's position at time $t$ into its jump is

$$x = x_0 - (V_{\text{leap}} \cos\phi_{\text{leap}})\,t \qquad \text{(Note sign!)}$$
$$y = y_0 + (V_{\text{leap}} \sin\phi_{\text{leap}})\,t - \tfrac{1}{2} g t^2.$$

The baby reaches $x = 0$ at time

$$t = \frac{x_0}{V_{\text{leap}} \cos\phi_{\text{leap}}} = \frac{x_0}{V_{\text{leap}}} \sec\phi_{\text{leap}},$$

at which moment

$$y = y_0 + x_0 \tan\phi_{\text{leap}} - \tfrac{1}{2} g \left(\frac{x_0}{V_{\text{leap}}} \sec\phi_{\text{leap}}\right)^2.$$

The baby lands on the wall if and only if $y(x=0) \geq 0$.

i.e., $$x_0 \tan\phi_{\text{leap}} - \frac{g}{2}\left(\frac{x_0}{V_{\text{leap}}} \sec\phi_{\text{leap}}\right)^2 \geq -y_0$$

$$V_{\text{leap}} \geq \sqrt{\frac{g x_0^2 \sec^2\phi_{\text{leap}}}{2(y_0 + x_0 \tan\phi_{\text{leap}})}} = 83.0\,\text{cm/s}$$

(Note: $g = 1{,}000\ \text{cm/s}^2$)

10. In the Wild Western Galaxy, a rocket train is moving along a straight track at a speed $v_t$ close to (but less than) the speed of light. Outlaw Oscar, standing at the front of the train, fires his gun (event 1). Oscar aims at Sheriff Stan, who is standing at the back of the train. The bullet misses Stan, but does knock off his hat (event 2).

    In the rest frame of the train, the length of the train is $L$ and the bullet's speed is $v_b$. Take the train's velocity as observed in the rest frame of the track to be directed along the $+x$ direction.

    (a) In the rest frame of the track, what is the time interval $t_2 - t_1$ between the two events?

    (b) In the rest frame of the track, what is the spatial displacement $x_2 - x_1$ between the two events?

    (c) Is it possible for the speed $v_b < c$ of the bullet to be chosen so that, in the rest frame of the track, it knocks off Stan's hat at the same spatial location as it is fired? If so, find the speed $v_b$ that achieves this.

    (d) Is it possible for the speed $v_b < c$ of the bullet to be chosen so that, in the rest frame of the track, it knocks off Stan's hat at the same time as it is fired? If so, find the speed $v_b$ that achieves this.

In the rest frame of the train...

$$\Delta x' \equiv x_2' - x_1' = -L \qquad \text{(Note sign!)}$$

$$\Delta t' = \frac{\Delta x'}{-V_b} = \frac{L}{V_b}.$$

In the rest frame of the track...

(a)

$$\Delta t = \gamma(\Delta t' + V_t \Delta x'/c^2) \quad \text{where } \gamma = \frac{1}{\sqrt{1+(V_t/c)^2}}$$

$$= \gamma\left[\frac{L}{V_b} + \frac{V_t(-L)}{c^2}\right]$$

$$= \frac{L}{V_b}\,\gamma\left(1 - \frac{V_t V_b}{c^2}\right).$$

(b)

$$\Delta x = \gamma(\Delta x' + V_t \Delta t')$$

$$= \gamma\left[(-L) + \frac{V_t L}{V_b}\right]$$

$$= -L\,\gamma\left(1 - \frac{V_t}{V_b}\right).$$

(c) $\Delta x = 0 \Rightarrow V_b = V_t < c \quad \Rightarrow$ It <u>is</u> possible.

(d) $\Delta t = 0 \Rightarrow V_b = \dfrac{c^2}{V_t} > c \quad \Rightarrow$ It <u>is</u> <u>not</u> possible.