PHY 2060 Spring 2007 — Exam 3

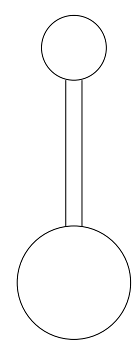
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all ten questions, each of which carries a maximum of 10 points. Write your solution below each question, continuing on additional paper if necessary. Please try to write neatly!

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. (This may benefit you even in the case of the multiple-choice Question 1.) Give all final algebraic answers in terms of variables defined in the problem, g (the acceleration due to gravity near the Earth's surface), and/or c (the speed of light). For numerical problems, take $g = 10\,\mathrm{m/s^2}$ and $c = 3.0 \times 10^8\,\mathrm{m/s}$.

During this exam, you may use three formula sheets and an electronic calculator. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any other electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

1. (a) A dog's toy consists of two uniform rubber spheres connected by a uniform rubber cylinder, whose axis lies along the line between the balls' centers (see figure). The cylinder's length is much greater than its diameter. The spheres have different diameters.
Let A be a rotation axis along the axis of the cylinder, B be a rotation axis perpendicular to the axis of the cylinder passing through the cylinder's center of mass, and C be a rotation axis parallel to B passing through the center of one of the spheres. Let I_X denote the rotational inertia of the toy about axis X.



Place a check to the left of any/all of the following statements that is/are **necessarily** true:

i. The rotational inertias satisfy $I_A > I_B$.
ii. The rotational inertias satisfy $I_B > I_A$.
iii. The rotational inertias satisfy $I_C > I_A$.
iv. The rotational inertias satisfy $I_C > I_B$.
v. The rotational inertias satisfy $I_B > I_C$.

(b) A particle of mass m has Cartesian coordinates (measured in meters) $x = -2t^2$, $y = 3$, $z = 0$, where $t \geq 0$ is the time (measured in seconds). The statements below concern the particle's angular momentum $\mathbf{l}$ about the origin $x = y = z = 0$.

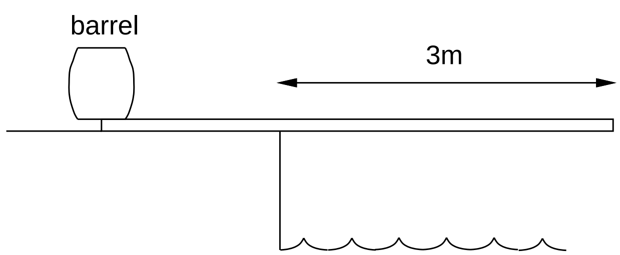
Place a check to the left of any/all of the statements that is/are **necessarily** true:

i. The angular momentum $\mathbf{l}$ has a magnitude that increases continuously from time $t = 0$.
ii. The angular momentum $\mathbf{l}$ has a magnitude that decreases continuously from time $t = 0$.
iii. The angular momentum $\mathbf{l}$ has a constant magnitude.
iv. The angular momentum $\mathbf{l}$ points along the negative z axis.
v. The angular momentum $\mathbf{l}$ points along the negative x axis.

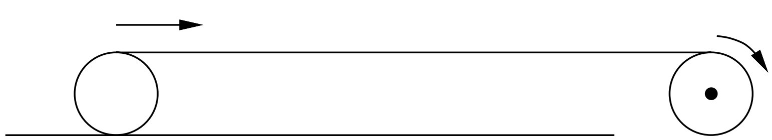
2. A car has wheels of diameter 70 cm. During the time that the car brakes to a complete stop from an initial speed of 45 km/h, the wheels complete 25 revolutions.

(a) What is the initial angular speed of the wheels? Give your answer in rad/s.
(b) What is the average angular acceleration of the wheels during the time the car is braking?

3. The crew of a pirate ship mutiny and vote to make their former captain walk the plank. A straight, uniform plank of mass 25 kg and length 5 m is placed on the flat deck of the ship so that 3 m of the plank extend overboard, as shown in the figure. A 100-kg barrel of water is placed on the plank at its inboard end to help anchor the plank in place. If the deposed captain has a mass of 80 kg, how close to the seaward end of the plank can he advance before he and the plank topple into the water?



4. A cylinder of radius 30 cm rests on a horizontal surface. An ideal string is partially wrapped around the cylinder, and the other end is wrapped around the rim of a wheel, also of radius 30 cm. The long segment of string between the cylinder and the wheel is taut and horizontal. At time $t = 0$, a drive motor begins to rotate the wheel with a constant angular acceleration of $0.2\,\text{rev/s}^2$. This rotation winds the string onto the wheel, and causes the cylinder to roll without slipping across the surface towards the wheel. How long does it take for the cylinder's center of mass to travel the first 3.5 m?



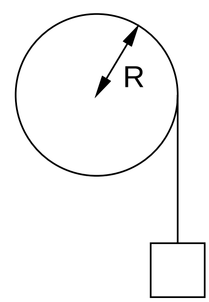
5. A barbell consists of a 2-m long, thin, uniform bar of mass 20 kg and two identical, circular plates whose thickness is negligible compared to their 40-cm radius. Each end of the bar is welded to the center of one of the plates, so that the bar is perpendicular to the plane of the plate. The rotational inertia of this barbell, when rotated about an axis perpendicular to the barbell passing through the center of one of the plates, is $231\,\mathrm{kg\,m^2}$.

   (a) Find the mass of each of the plates.

   (b) Calculate the rotational inertia of the barbell about an axis perpendicular to the bar passing through the bar's center.

6. A thin, circular plate has radius r, uniform thickness t, and uniform density ρ. A square hole, with sides of length w, is cut from the plate. The square lies entirely inside the circular perimeter, with its center located a distance d from the center of the circle. Find the rotational inertia of the plate about an axis perpendicular to the plane of the plate, passing through the point on the outer rim of the plate closest to the center of the hole.

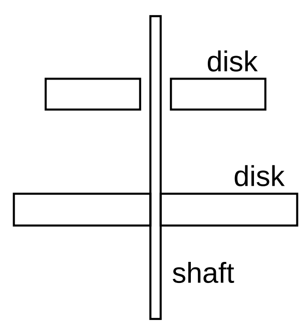
7. A solid, uniform sphere of mass M and radius R pivots on a fixed, massless, frictionless axle that passes through the center of the sphere. An ideal string is partially wrapped around the sphere's equator so that the sphere rotates when a mass $M/2$ is hung from the other end of the string.



   (a) What is the mass' acceleration just after this system is released from rest?

   (b) What is the angular momentum of the system about the axle at time t after the system was released from rest? (Assume that at time t, the string has not yet fully unwound from the sphere.)

8. The lower disk in the figure has mass 240 g and radius 3.5 cm. It is initially rotating at 80 rev/min on a light, frictionless shaft of negligible radius. The upper disk, of mass 110 g and radius 3.0 cm, is at rest. It is allowed to drop freely down the shaft onto the lower disk, and frictional forces act to bring the two disks to a common angular velocity. The disks are of uniform thickness and density.



   (a) What is the common angular velocity of the two disks?

   (b) What is the magnitude of the constant torque (measured about the shaft) that would be required to bring the two disks to rest in a time of 20 s?

9. A uniform meter stick can rotate freely about an axle passing through a hole bored at the 75-cm mark. The stick is held in a horizontal position and then released.

   (a) What is the stick's angular acceleration immediately after it is released?

   (b) What is the magnitude and direction of the force exerted by the axle on the meter stick immediately after the stick is released?

10. A ladder has mass M, length L, and can be treated as having a uniform mass per unit length. The ladder leans against a flat, frictionless wall that makes an angle ϕ with the vertical as shown in the figure. The coefficient of static friction between the ladder and the horizontal ground is μ_s. What is the minimum angle θ between the ladder and the ground that will ensure that the ladder remains stable? You should aim to get a final result of the form $\theta = \text{atan}(\ldots)$, where "…" does not depend on θ.