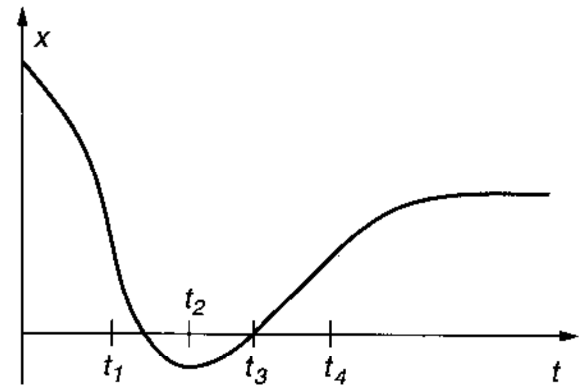
PHY 2060 Spring 2007 – Exam 1 Solution

1. (a) The graph below shows the position x versus time t for an object moving in one dimension.



Place a check to the left of any/all of the following statements that is/are true:

i. The object has its maximum speed between times t_3 and t_4.

ii. The object's acceleration is greatest in magnitude near time t_1.

✓ iii. The object's acceleration is approximately constant between times t_3 and t_4.

iv. The object is stationary at time t_3.

The object's velocity is $v = \frac{dx}{dt}$ (the slope of x vs t)

Its acceleration is $a = \frac{d^2x}{dt^2}$ (the curvature of x vs t)

i. <u>False</u> The slope has greatest magnitude near $t = t_1$.

ii. <u>False</u> The curvature has greatest magnitude near $t = t_2$. In fact there is a point of inflection (corresponding to zero acceleration) near $t = t_1$.

iii. <u>True</u> The curve is nearly straight between t_3 and t_4, corresponding to constant velocity and constant (zero) acceleration.

iv. <u>False</u> The slope (and hence the speed) is nonzero at $t = t_3$.

(b) A projectile is launched at an angle of 45° above horizontal ground. During its flight, the projectile experiences drag due to air resistance. Place a check to the left of any/all of the following statements that is/are true:

i. The object instantaneously experiences zero acceleration at the highest point of its trajectory.

✓ ii. The horizontal component of the object's velocity is decreasing in magnitude at every moment of the flight.

✓ iii. The object travels a greater horizontal distance during the first half of its flight time than it does during the second half.

iv. The vertical component of the object's velocity is decreasing in magnitude at every moment of the flight.

v. The vertical component of the object's acceleration is constant.

vi. The object instantaneously experiences zero drag at the highest point of its trajectory.

At each moment, the projectile experiences two forces:

- Its weight, vertically downward.
- A drag force in the opposite direction to the projectile's instantaneous velocity. The magnitude of the drag force increases with the projectile's speed.

i. <u>False</u> At the highest point, $V_y = 0$. The object experiences its weight and a horizontal drag force.

ii. <u>True</u> The object's acceleration always has a horizontal component opposite in sign to V_x.

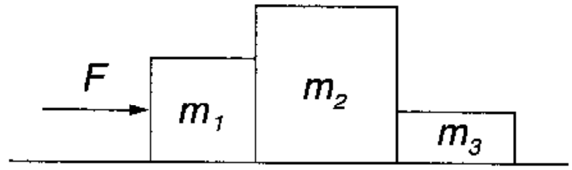
iii. <u>True</u> Since $|dV_x/dt| < 0$, $\left|\int_0^{T/2} V_x(t)dt\right| > \left|\int_{T/2}^{T} V_x(t)dt\right|$.

iv. <u>False</u> At the highest point, $|V_y| = 0$. However, $|V_y| > 0$ both before and after this moment.

v. <u>False</u> The weight is constant but the vertical component of the drag changes in time, and hence so does a_y.

vi. <u>False</u> See the answer to i above.

2. Three blocks are in contact on a horizontal, frictionless table. A force of magnitude F is applied to the left-most block, as shown below. Find the magnitude and direction of the force exerted by the rightmost block on the middle block.



- First, let us treat the three blocks as a single body.

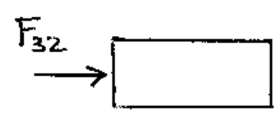
  N2 applied in the horizontal direction gives

$$F = (m_1 + m_2 + m_3)\,a,$$

  so each block experiences an acceleration

$$a = \frac{F}{m_1 + m_2 + m_3}$$

- Now consider block 3 alone:



  Since the only horizontal force is F_{32},

  N2 gives

$$F_{32} = m_3 a = \frac{m_3}{m_1 + m_2 + m_3} F$$

- Finally, N3 tells us that the force exerted on block 2 by block 3 is

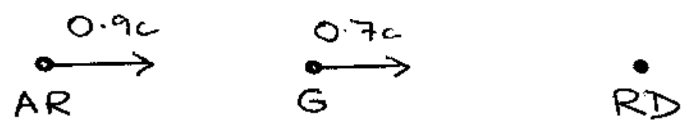
$$F_{23} = -F_{32}.$$

Thus, the rightmost block exerts a force of magnitude

$\frac{m_3}{m_1 + m_2 + m_3} F$ to the left on the middle block.

3. The crew of the Jupiter Mining Vessel *Red Dwarf* observe a GELF (genetically engineered lifeform) cruiser approaching at $0.7c$. Fearless superhero Ace Rimmer comes to the rescue in *Starbug*, overtaking the GELFs from behind at a speed of 0.9c relative to *Red Dwarf*. At what speed do the GELFs observe Ace approaching?

- Let S be the rest frame of Red Dwarf:



In this frame, Ace Rimmer has a velocity

$$V_x = 0.9c$$

- Let S' be the rest frame of the GELFs, who move at velocity

$$v = 0.7c$$

along the x axis of frame S.

The Lorentz velocity addition formula gives

$$V_x' = \frac{V_x - v}{1 - vV_x/c^2}$$

$$= \frac{0.9c - 0.7c}{1-(0.7)(0.9)}$$

$$= 0.54c$$

This is the speed at which the GELFs observe Ace Rimmer approaching

4. A spaceship, at rest, has a length of 50 m. A hangar has a length of 40 m.

   (a) How fast must the spaceship be moving relative to the hangar in order to fit inside (according to an observer standing next to the hanger)?

   (b) When traveling at the speed you determined in (a), will the pilot of the spaceship agree that the spaceship fits into the hangar? Don't just answer "yes" or "no"; support your conclusion with hard numbers.

(a) When moving at speed v, the spaceship of rest length L_{so} will be observed to be length-contracted to

$$L_s = \frac{L_{so}}{\gamma} = L_{so}\sqrt{1-(v/c)^2}$$

To fit inside the hangar (as observed in the hangar's rest frame), the ship's length must be

$$L_s = L_{ho}$$

$$L_{so}\sqrt{1-(v/c)^2} = L_{ho}$$

$$\frac{v}{c} = \sqrt{1-\left(\frac{L_{ho}}{L_{so}}\right)^2}$$

This yields $\quad v = 0.6c$

(b) The spaceship pilot will observe the ship to have its rest length

$$L_{so} = 50\,\text{m}$$

but the hangar will be length-contracted to

$$L_h = \frac{L_{ho}}{\gamma} = L_{ho}\sqrt{1-(v/c)^2}$$

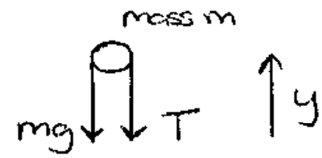
$$= \frac{L_{ho}^2}{L_{so}}$$

$$= 32\,\text{m}$$

The pilot will conclude that the ship does not fit.

(The resolution of this "paradox" lies in the different notions of simultaneity in the two reference frames.)

5. A 50-kg girl stands on a scale placed on the ground. The girl holds a massless string, which she uses to swing a 3.0-kg mass around in a vertical circle of radius 60 cm at a constant rate of 0.8 rev/sec. What is the reading on the scale (in newtons) at the moment the mass is at the highest point in its path?



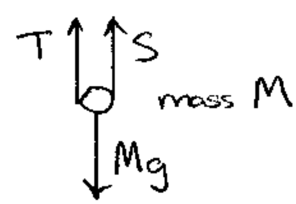
Mass : At the highest point in its path, the mass experiences both its weight mg and the string tension T acting vertically downwards.

Newton's 2nd law in the y direction gives

$$-mg - T = ma_{m,y} = -\frac{mv^2}{r} \quad \text{(centripetal acceleration)}$$

where v is the velocity and r is the radius.

Thus, $$T = m\left(\frac{v^2}{r} - g\right).$$



Girl : When the mass is at its highest point, the girl experiences both the string tension T and the contact force of the scale S acting vertically upward.

Newton's 2nd law gives

$$T + S - Mg = Ma_g = 0$$

$$S = Mg - T$$

$$= (M+m)g - \frac{mv^2}{r}$$

To evaluate S, we must find

$$v = (\text{circumference of circle}) \times (\text{revs/sec})$$

$$= 2\pi(0.60\,\text{m})(0.8\,\text{rev/sec})$$

$$= 3.02\,\text{m/s}$$

$\Rightarrow$ $$T = 15.5\,\text{N}$$

and $$S = 485\,\text{N}$$

6. Dr. Naughty, anxious to be rid forever of his nemesis Captain Improbable, straps him into a rocket with a bomb set on a 1-year fuse, lights the fuse, and immediately launches the rocket away from Earth at a speed of $0.6c$. A camera beams back pictures of the inside of the rocket back to Earth so that Dr. Naughty can gloat over our hero's fate. Like all electromagnetic waves, the TV signals travel through outer space at speed c.

   (a) How far from Earth—as measured in the Earth's reference frame—will the rocket be when the fuse completes its 1-year burn time?

   (b) How long after the rocket's launch—as measured on Earth—does Dr. Naughty receive the signal showing Captain Improbable, who has gnawed through his restraints, disarming the bomb the instant before it is about to explode?

   (c) Within moments of disarming the bomb, Captain Improbable rigs up his iPod to communicate with the navigation computer, turns the rocket round, and sets off back to Earth at $0.6c$. How long after his escape—as measured in the frame of the rocket—does Captain Improbable reach Dr. Naughty's hideout to settle old scores?

(a) Due to time dilation, one year's elapsed time on the rocket is observed on Earth to correspond to

$$\Delta t = \gamma \Delta t' = \frac{\Delta t'}{\sqrt{1-(v/c)^2}}$$

$$= 1.25 \text{ yr}$$

$\Rightarrow$ Distance travelled by rocket

$$d = v\Delta t = (0.6c)(1.25 \text{ yr})$$
$$= 0.75 \text{ light yr}$$

(b) Allowing for the transmission time of the TV signal, the time on Earth to the arrival of the pictures is

$$= 1.25 \text{ yr} + \frac{0.75 \text{ light yr}}{c}$$
$$= 2.0 \text{ yr}$$

(c) In the rocket's frame, the time for the return journey is the same as the time for the outward journey,

$$\Delta t' = 1 \text{ yr}$$

7. A golfer strikes a ball from ground level at an angle of 20° above the horizontal with an initial speed of 70 m/s. The ball misses the fairway and bounces off a flat road. The horizontal component of the ball's velocity is conserved during the bounce. After the bounce, the ball reaches one third of the height it reached on its first arc before landing in a sand trap. Assume that the launch point and the two landing points are all at the same elevation, and neglect air resistance.

   (a) What is the maximum height reached by the ball on its first arc?

   (b) What is the distance from the ball's launch point to the location of its bounce?

   (c) How far from the launch point does the ball hit the sand trap?

Let us take the ground level to be $y=0$.

(a) We can find the maximum height y_{max} for the first arc by setting $v_y = 0$ in the standard equation

$$v_y^2 = v_{oy}^2 - 2gy$$

$$y_{max} = \frac{v_{oy}^2}{2g} \quad (1)$$

$$= \frac{(v_0 \sin\phi_0)^2}{2g} = \frac{[(70\,m/s)(\sin 20^\circ)]^2}{2\,(10\,m/s^2)}$$

$$= 28.7\,m$$

(b) The distance is given by the horizontal range formula

$$R = \frac{v_0^2}{g}\sin 2\phi_0 \quad (2)$$

$$= \frac{(70\,m/s)^2}{10\,m/s^2}\sin 40^\circ$$

$$= 315\,m$$

(c) The ball's second arc is projectile motion with an initial velocity

$$v'_{ox} = v_{ox}$$

and

$$v'_{oy} = \sqrt{2g\,y'_{max}} \qquad \text{from } (1)$$

$$y'_{max} = y_{max}/3 \Rightarrow v'_{oy} = \sqrt{2g\,y_{max}/3} = v_{oy}/\sqrt{3}$$

We can now find v'_0 and ϕ'_0, and hence R'. Alternatively, we can recognize that (2) is equivalent to

$$R = \frac{2v_0^2}{g}\sin\phi_0\cos\phi_0 = \frac{2v_{ox}v_{oy}}{g}$$

Thus

$$R' = \frac{2v'_{ox}v'_{oy}}{g} = R/\sqrt{3}$$

$\Rightarrow$ Total distance, $R + R' = 497\,m$

8. Typically, a car braking hard on a wet road can safely decelerate at $3.0\,\mathrm{m/s^2}$. The driver's reaction time to engage the brakes may be as great as 1.5 s. Given this information, what is the maximum speed at which cars can safely drive 60 m apart in the same lane, allowing for a worst-case scenario in which the lead car unexpectedly hits an immovable object and stops instantaneously?

If the driver's reaction time is t_r, then a car moving at speed V_0 travels a distance

$$d_1 = V_0 t_r$$

before the brakes are applied.

Once the brakes are applied, the additional stopping distance d_2 is given by

$$0 = v^2 = V_0^2 + 2ad_2$$

$$d_2 = \frac{V_0^2}{2|a|}$$

The total stopping distance is

$$d = d_1 + d_2 = V_0 t_r + \frac{V_0^2}{2|a|}$$

Solve for V_0:

$$\tfrac{1}{2}V_0^2 + |a|t_r V_0 - |a|d = 0$$

$$V_0 = -|a|t_r + \sqrt{(|a|t_r)^2 + 2|a|d}$$

$$= -(3.0\,\mathrm{m/s^2})(1.5\,\mathrm{s}) + \left[(3.0\,\mathrm{m/s^2})^2(1.5\,\mathrm{s})^2 + 2(3.0\,\mathrm{m/s^2})(60\,\mathrm{m})\right]^{\frac{1}{2}}$$

$$= 15\,\mathrm{m/s} = 54\,\mathrm{km/h} = 34\,\mathrm{mph}$$

9. In a circus show, a large pot (initially at rest) is dropped from a point at height h. At the same moment, a clown standing directly below throws a ball vertically upward from the ground. The ball strikes the pot head-on, smashing it into pieces and thereby saving the clown from severe concussion (or worse). At the moment of impact, the ball's speed equals that of the pot. At what height does the collision occur? Neglect air resistance.

Let us measure y vertically upward from grond level. Then the pot ("p") and the ball ("b") obey

$$y_p = h - \tfrac{1}{2}gt^2 \qquad v_p = -gt$$

$$y_b = v_0 t - \tfrac{1}{2}gt^2 \qquad v_b = v_0 - gt$$

The collision occurs at time t_c and height y_c such that

$$y_c = y_p(t_c) = y_b(t_c)$$

$$\Rightarrow \quad h = v_0 t_c \qquad \text{or} \qquad t_c = \frac{h}{v_0} \qquad (1)$$

We are also told that

$$-v_p(t_c) = v_b(t_c)$$

$$\Rightarrow \quad g t_c = v_0 - g t_c \qquad \text{or} \qquad t_c = \frac{v_0}{2g} \qquad (2)$$

(1) and (2) are consistent only if

$$\frac{h}{v_0} = \frac{v_0}{2g} \qquad \text{or} \qquad v_0^2 = 2gh \qquad (3)$$

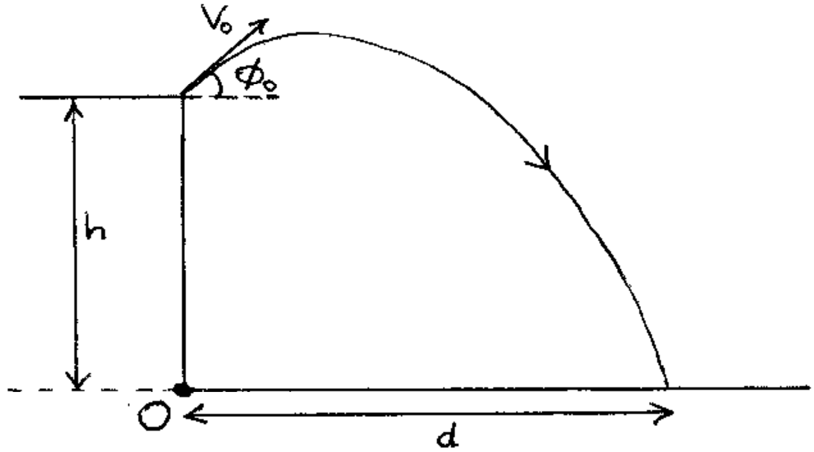
Substituting (1) and (3) into the y_p equation gives

$$y_c = h - \tfrac{1}{2}g\,\frac{h^2}{2gh}$$

$$= \frac{3h}{4}$$

10. Standing on the edge of the Grand Canyon, you wish to fire an arrow carrying a message to some friends, who are located on a rock outcrop located 800 m below your position and a distance 500 m away horizontally. If the arrow has an initial speed of 40 m/s, at what angle to the horizontal (specify above or below) should you launch it? Neglect air resistance.

Hints: (1) There are two possible correct answers to this problem. (2) It may be helpful to recall the trigonometric identity $1/\cos^2 x = 1 + \tan^2 x$.



Taking the origin to be the point O, the path followed by the arrow is (based on the lecture notes)

$$y = h + x \tan\phi_0 - \frac{1}{2} g \left(\frac{x}{V_0 \cos\phi_0}\right)^2$$

In order for the arrow to land at $x = d$, we require

$$\begin{aligned} 0 &= h + d\tan\phi_0 - \frac{1}{2} g \left(\frac{d}{V_0\cos\phi_0}\right)^2 \\ &= h + d\tan\phi_0 - \frac{gd^2}{2V_0^2}(1+\tan^2\phi_0) \\ &= (h-c) + d\tan\phi_0 - c\tan^2\phi_0 \end{aligned}$$

where $\quad c = \frac{gd^2}{2V_0^2}.$

$$\tan\phi_0 = \frac{-d \pm \sqrt{d^2 + 4c(h-c)}}{-2c} = \frac{d}{2c} \pm \sqrt{\left(\frac{d}{2c}\right)^2 + \frac{h}{c} - 1}$$

Substituting $d = 500\,\text{m}$, $h = 800\,\text{m}$, $V_0 = 40\,\text{m/s}$, $g = 10\,\text{m/s}^2$,

$$\tan\phi_0 = 0.676 \text{ or } -0.036$$

$$\phi_0 = \begin{cases} +34.0^\circ & \text{(above the horizontal)} \\ -2.0^\circ & \text{(below the horizontal)} \end{cases}$$