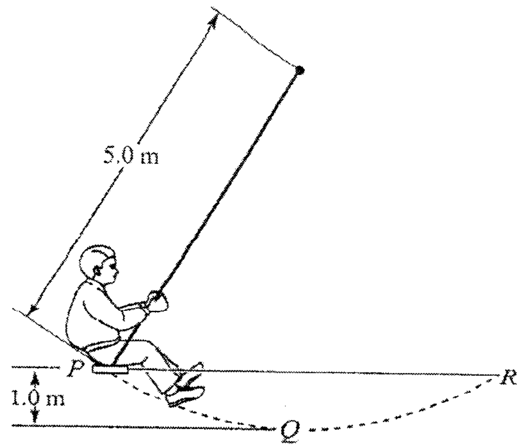
PHY 2060 Spring 2007 – Final Exam Solution

1. (a) P and R mark the highest and Q the lowest positions of a 50.0-kg boy swinging as illustrated in the figure. What is the tension in the rope at point Q?



____ 1. 250 N

____ 2. 525 N

✓ 3. 7×10^2 N

____ 4. 1.1×10^3 N

____ 5. None of the above.

N2 along y ↑ : $T - mg = ma$

$= \frac{mv^2}{r}$ centripetal force

Conservation of mechanical energy

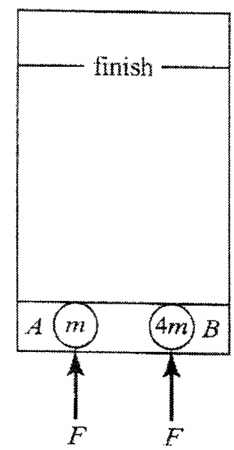
$\frac{1}{2}mv^2 = mgh$

$\Rightarrow$ $T = mg\left(1 + \frac{2h}{r}\right)$

$= (50.0\,\text{kg})(10\,\text{m/s}^2)\left[1 + \frac{2(1.0\,\text{m})}{5.0\,\text{m}}\right]$

$= 700\,\text{N}.$

(b) The diagram depicts two pucks on a frictionless table. Puck B is four times as massive as puck A. Starting from rest, the pucks are pushed across the table by two *equal* forces.



Which puck has the greater kinetic energy upon reaching the finish line?

____ 1. Puck A

____ 2. Puck B

_✓_ 3. They both have the same kinetic energy.

____ 4. Too little information to answer

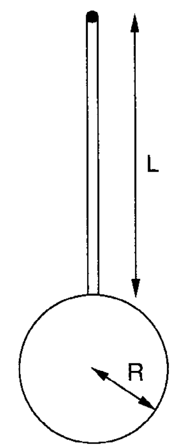
Work-energy theorem:

$$\Delta K = W_{ext}$$

$$\Rightarrow \quad K_f = Fs,$$

which is the same for A and B.

2. A pendulum consists of a solid, uniform sphere of mass M and radius R attached to one end of a thin, uniform rod of mass m and length L. The pendulum swings freely about the other end of the rod. Find the period of small oscillations of this pendulum.



Physical pendulum: $T = 2\pi\sqrt{\frac{I}{mgd}}$

Here $I = I_{rod} + I_{sphere}$

$I_{rod} = \frac{1}{3}mL^2$

$I_{sphere} = \frac{2}{5}MR^2 + M(L+R)^2$ parallel-axis theorem

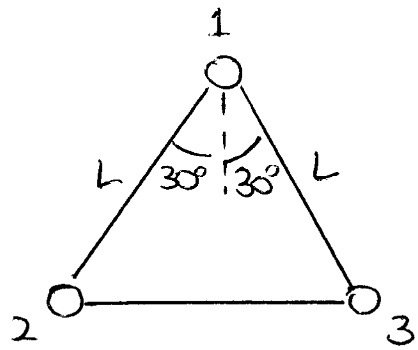
and "md" $= m\frac{L}{2} + M(L+R)$

$\Rightarrow$ $T = 2\pi\sqrt{\frac{2}{15}\,\frac{5mL^2 + 3M(5L^2 + 6LR + 7R^2)}{mL + 2M(L+R)}}$

3. Three dense spheres, each of mass M, are located at the corners of an equilateral triangle of sides L. All other bodies in the universe are far enough away that their gravitational influence can be neglected.

   (a) Find the magnitude of the gravitational force exerted on one of the spheres.

   (b) What minimum amount of work must be done by external forces to move the masses off to infinite separation from one another?



(a) For 1, say, horizontal components of forces due to 2 and 3 cancel, while vertical components add:

$$F_{net} = \frac{GM^2}{L^2} 2\cos 30°$$

$$= \frac{\sqrt{3}GM^2}{L^2}$$

(b) Minimum work to take masses to infinity

$$= \text{binding energy}$$

$$= -U_{12} - U_{13} - U_{23}$$

$$= 3\frac{GM^2}{L}$$

4. (a) Amy throws a ball vertically downward with an initial speed of 10 m/s from the top of the Century Tower. How long does the ball take to reach the ground, 48 m below?

(b) Bob throws a ball vertically upward from ground level so that it just reaches Amy on top of the Century Tower. How fast does Bob throw the ball?

(c) Now Amy and Bob repeat their throws simultaneously, and the balls collide in the air. How long after they throw does this collision occur?

(a)

$$y = y_0 + v_0 t - \tfrac{1}{2} g t^2$$
$$t = \frac{1}{g}\left[v_0 \pm \sqrt{v_0^2 + 2g(y_0 - y)}\,\right]$$

Here, $v_0 = -10\,\text{m/s}$, $y_0 - y = 48\,\text{m}$:

$\Rightarrow \qquad t = 2.26\,\text{s}$

(b)

$$v^2 = v_0^2 - 2g(y - y_0)$$
$$v_0 = \pm\sqrt{v^2 + 2g(y - y_0)}$$

We need the positive root with $v = 0$, $y - y_0 = 48\,\text{m}$:

$\Rightarrow \qquad v_0 = 31.0\,\text{m/s}$

(c) Amy's ball: $\quad y_A = y_{A0} + v_{A0} t - \tfrac{1}{2} g t^2$

Bob's ball: $\quad y_B = y_{B0} + v_{B0} t - \tfrac{1}{2} g t^2$

$$y_A - y_B = y_{A0} - y_{B0} + (v_{A0} - v_{B0})t$$
$$t = \frac{(y_{A0} - y_{B0}) - (y_A - y_B)}{v_{B0} - v_{A0}}$$

Here $y_{A0} - y_{B0} = 48\,\text{m}$, $y_A - y_B = 0$, $v_{B0} - v_{A0} = 41.0\,\text{m/s}$:

$\Rightarrow \qquad t = 1.17\,\text{s}$

5. You apply a horizontal force of 40 N to a box of mass 2.5 kg, initially at rest on a horizontal surface. Applying this force, you slide the box through a distance of 7.0 m. The coefficient of kinetic friction between the box and the table is 0.30.

   (a) How much work do you perform on the box?

   (b) How fast is the box traveling at the end of the 7.0 m?

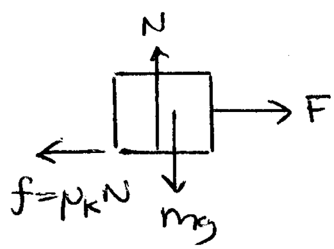
(a)

$$W = \int \vec{F} \cdot d\vec{s}$$

$$= F s \qquad \text{here}$$

$$= (40\,\text{N})(7.0\,\text{m})$$

$$= 280\,\text{J}$$



(b) Applying N2 horizontally,

$$F - f = ma$$

$$a = \frac{F-f}{m}$$

$$v^2 = \cancel{v_0^2}^{\,0} + 2as = 2\,\frac{F-f}{m}\,s$$

$$= 2\left(\frac{F}{m} - \mu_k g\right)s$$

$$v = \sqrt{2\left(\frac{F}{m} - \mu_k g\right)s}$$

6. Greg and Jane, of mass m_G and m_J, respectively, stand at opposite ends of a cart of mass m_c and length L. The cart is free to roll without friction on a horizontal track. Initially the cart is at rest, and Greg holds a pumpkin of mass m_p.

   (a) On a whim, Greg throws the pumpkin towards Jane in such a manner that the horizontal component of the pumpkin's initial velocity relative to the ground is $\mathbf{v}_p$. Find the velocity of the cart relative to the ground immediately after the pumpkin is thrown.

   (b) How long does it take the pumpkin to reach Jane?

   (c) Find the velocity of the cart relative to the ground immediately after Jane catches the pumpkin.

(a) By conservation of linear momentum

$$m_p\vec{v}_p + (m_c + m_G + m_J)\vec{v}_c = \vec{0}$$

$$\vec{v}_c = -\frac{m_p}{m_c + m_G + m_J}\vec{v}_p$$

(b) Pumpkin's velocity relative to Jane is

$$\vec{v}_p' = \vec{v}_p - \vec{v}_c$$

$$= \left(1 + \frac{m_p}{m_c + m_G + m_J}\right)\vec{v}_p$$

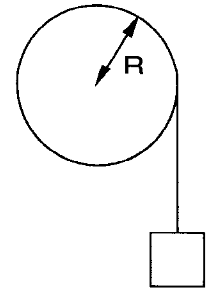
$$t = \frac{L}{|\vec{v}_p'|}$$

$$= \frac{m_c + m_G + m_J}{m_c + m_G + m_J + m_p}\frac{L}{|\vec{v}_p|}$$

(c) Since the system's total momentum is zero,

$$\vec{v}_{final} = \vec{0}.$$

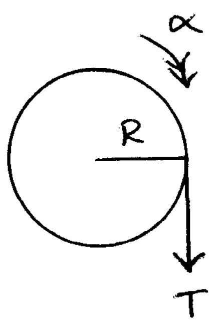
7. A solid, uniform cylinder of mass M and radius R pivots on a fixed, massless, frictionless axle that lies along the cylinder's axis of rotational symmetry. An ideal string is partially wrapped around the cylinder so that the cylinder rotates when a mass $2M$ is hung from the other end of the string.

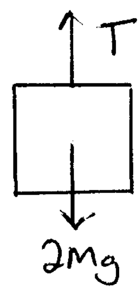


(a) What is the angular acceleration of this system just after it is released from rest?

(b) What is the angular momentum of the system about the axle at time t after the system was released from rest? (Assume that at time t, the string has not yet fully unwound from the cylinder.)

(a)





$\downarrow$ $a = R\alpha$

N2:

$$TR = \tfrac{1}{2}MR^2\alpha \qquad T - 2Mg = -2Ma$$

$$T = \tfrac{1}{2}Ma \longrightarrow \tfrac{1}{2}Ma - 2Mg = -2Ma$$

$$a = \frac{4g}{5}$$

$$\alpha = \frac{4g}{5R} \quad \text{(clockwise)}$$

(b) Since

$$\tau_{net} = \frac{dL}{dt}$$

$$L = \int_0^t \tau_{net}\, dt'$$

$$= \underbrace{2MgR}_{\text{torque due to gravity}}\ t$$

8. Two spaceships, each 80 m long when measured at rest, travel towards one other at equal speeds of $0.9c$ as measured on Earth.

(a) How long is each spaceship, as measured on Earth?

(b) How long does an observer on spaceship 1 measure spaceship 2 to be?

(a) An observer on Earth measures a length-contracted value

$$L_E = \frac{L_0}{\gamma_V} = L_0\sqrt{1-(V/c)^2}$$

$$= (80\,m)\sqrt{1-(0.9)^2}$$

$$\simeq 35\,m$$

(b) The velocity of ship 2 measured on ship 1 is

$$V_x' = \frac{V_x - U}{1 - UV_x/c^2}$$

$$= \frac{-0.9c - 0.9c}{1-(0.9)(-0.9)}$$

$$\approx 0.9945\,c$$

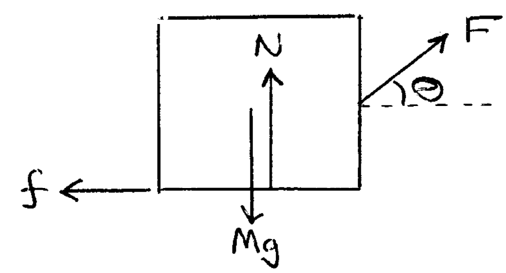
$$L' = \frac{L_0}{\gamma_{V'}} = L_0\sqrt{1-(V'/c)^2}$$

$$\approx 8.4\,m$$

9. You want to pull a box of mass M over a horizontal floor by using a massless rope tied to the box. The coefficients of static and kinetic friction between the box and the floor are μ_s and μ_k, respectively. Having taken PHY 2060, you realize that it may be to your advantage to pull on the rope (and hence on the box) not horizontally, but rather at an angle θ above the horizontal.

   (a) The box is initially at rest. What angle θ will minimize the force that you must apply to start the box moving?

   (b) What angle θ will minimize the work that you do in moving the box a horizontal distance s?



(a) N2 along x: $F\cos\theta - f = Ma_x \simeq 0$

y: $F\sin\theta + N - mg = Ma_y \simeq 0$

Right before the box starts moving, the friction force satisfies

$$f = \mu_s N = \mu_s(Mg - F\sin\theta)$$

$$\Rightarrow \quad F\cos\theta = \mu_s(Mg - F\sin\theta)$$

$$F = \frac{\mu_s Mg}{\cos\theta + \mu_s \sin\theta}$$

Minimize:

$$0 = \frac{dF}{d\theta} = \frac{-\mu_s Mg}{(\cos\theta + \mu_s\sin\theta)^2}(-\sin\theta + \mu_s\cos\theta)$$

$$\theta = \tan^{-1}\mu_s$$

(b) Work done $\quad W = \int \vec{F}\cdot d\vec{s}$

If lift box infinitesimally off the floor,

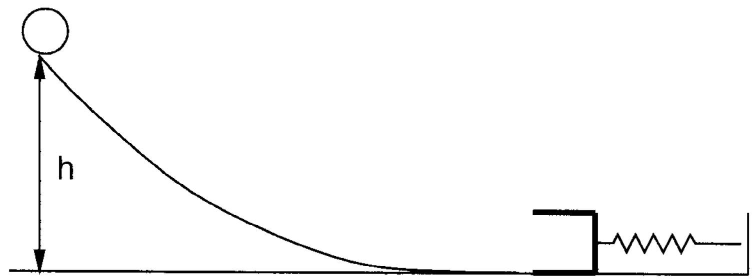
$$F_x = 0, \quad F_y = Mg$$

$$\Rightarrow \quad W = 0$$

So minimize work by choosing

$$\theta = 90^\circ$$

10. A uniform, spherical ball of mass m and radius r is released from rest on a track at a point at height h above the ground. The ball rolls without slipping down to ground level, where it buries itself in a padded box of mass M that has been set up at the end of the track. The coefficient of kinetic friction between the box and the ground is μ_k. The box is attached (as shown in the diagram) to a horizontal, ideal spring of spring constant k. The box with the ball inside compresses the spring a maximum distance d from its initial, unstretched length. Find an expression for d in terms of g and other variables defined in the problem.



1. The roll downhill conserves mechanical energy. Speed v_1 of ball right before it hits the box is given by

$$\Delta K + \Delta U = 0$$
$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}I\omega_1^2 = mgh$$
$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)(v_1/r)^2 = mgh$$
$$v_1 = \sqrt{\frac{10gh}{7}}$$

2. Totally inelastic collision conserves linear momentum. Common speed V_2 of ball and box immediately after collision obeys

$$(M+m)V_2 = mv_1$$
$$V_2 = \frac{m}{M+m}\sqrt{\frac{10gh}{7}}$$

3. Compression of spring is accompanied by dissipation of mechanical energy due to friction:

$$\Delta K + \Delta U = W_f - \Delta E_{int} = -fd$$
$$-\tfrac{1}{2}(M+m)V_2^2 + \tfrac{1}{2}kd^2 = -\mu_k(M+m)gd \quad \leftarrow \text{quadratic in } d$$

$$d = \frac{1}{k}\left\{-\mu_k(M+m)g + \sqrt{[\mu_k(M+m)g]^2 + k(M+m)V_2^2}\right\}$$
$$= \frac{1}{k}\left\{\sqrt{[\mu_k(M+m)g]^2 + \frac{10m^2gh}{7(M+m)}} - \mu_k(M+m)g\right\}$$