PHY 2060 Spring 2008 – Exam 1 Solution

1. A mass m is thrown vertically upward from height $y = 0$ with an initial velocity v_0. Between the time of its release and the time it returns to $y = 0$, the mass is acted on by two forces: its weight mg, and a drag force of magnitude md (where d has dimensions of acceleration, and satisfies $0 < d < g$). At each moment, the drag force is directed in the direction opposite to the particle's instantaneous velocity.

   Place a check to the left of any/all of the following statements that is/are necessarily true:

   ✗ i. The time for the upward leg of the particle's motion is greater than the time for the downward motion back to $y = 0$.

   ✓ ii. At any height y above the launch height, the particle's speed is lower than it would be in the absence of the drag force (for the same value of v_0).

   ✗ iii. The total time for the particle's round trip to/from $y = 0$ is greater than it would be in the absence of the drag force (for the same value of v_0).

   ✗ iv. The particle's speed when it passes any given height y above the launch point is the same on the upward leg as on the downward leg.

   ✓ v. The maximum height y reached during the particle's motion is less than it would be in the absence of the drag force (for the same value of v_0).

<u>On way up</u> acceleration $a = -(g+d)$

velocity $v = v_0 - (g+d)t$

$\Rightarrow$ Reaches top after time $t_{up} = \frac{v_0}{g+d}$

Also, $v^2 = v_0^2 - 2(g+d)y$ consistent with (ii)

$\Rightarrow$ max. height reached, $h = \frac{v_0^2}{2(g+d)}$ confirms (v)

<u>On way down</u> $a = -(g-d)$

Measuring t from top, $y = h - \frac{1}{2}(g-d)t^2$

$\Rightarrow$ Reaches $y=0$ after time $t_{down} = \sqrt{\frac{2h}{g-d}}$

$= \frac{v_0}{\sqrt{g^2-d^2}} > t_{up}$ disproves (i)

Also

$$v^2 = 2(g-d)(h-y)$$
$$= \frac{g-d}{g+d}v_0^2 - 2(g-d)y$$
$$= (v_0^2 - 2gy) - 2d(2h-y)$$

consistent with (ii), disproves (iv)

<u>Round trip time</u> $t_{up} + t_{down} = \frac{v_0}{g+d} + \frac{v_0}{\sqrt{g^2-d^2}}$

$< \frac{2v_0}{g}$ for $d \ll g$ disproves (iii)

$> \frac{2v_0}{g}$ for $d \to g$

2. The coordinates x and y (in meters) of a particle as a function of time t (in seconds) are $x = 3t^2$ and $y = 16t - 4t^2$.

   (a) Find the particle's velocity and acceleration at time t.

   (b) Find the particle's smallest speed.

(a) Since $x = 3t^2$ $\quad$ $y = 16t - 4t^2$;

$$\Rightarrow \quad V_x = \frac{dx}{dt} = 6t \qquad V_y = \frac{dy}{dt} = 16 - 8t$$

and

$$a_x = \frac{dV_x}{dt} = 6 \text{ m/s}^2 \qquad a_y = \frac{dV_y}{dt} = -8 \text{ m/s}^2$$

(b) Particle's speed is

$$V = \sqrt{V_x^2 + V_y^2}$$
$$= \sqrt{(6t)^2 + (16-8t)^2}$$
$$= \sqrt{100t^2 - 256t + 256}$$

$$\frac{dv}{dt} = \frac{1}{2v}(200t - 256)$$

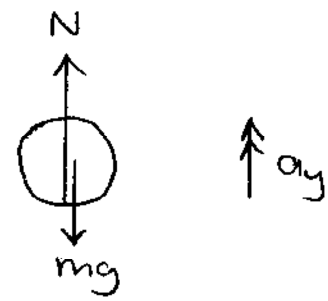
To minimize v, look for $\frac{dv}{dt} = 0$:

$$t = \frac{256}{200} = 1.28 \text{ s}$$

$$v = 9.6 \text{ m/s}$$

3. A man of 50 kg stands on a bathroom scale placed on the floor of an elevator. What does the scale read when the elevator is ...

   (a) stationary?
   (b) accelerating downwards at $2.0\,\mathrm{m/s^2}$?
   (c) moving downwards with a constant velocity of $6.0\,\mathrm{m/s}$?
   (d) slowing uniformly to rest from a downwards velocity of $6.0\,\mathrm{m/s}$ over the course of $4.0\,\mathrm{s}$?



N2 resolved along the vertical direction gives

$$N - mg = ma_y$$

The scale reading is

$$N = m(g + a_y).$$

(a) $a_y = 0 \Rightarrow$ $N = (50\,\mathrm{kg})(10\,\mathrm{m/s^2}) = 500\,\mathrm{N}.$

(b) $a_y = -2.0\,\mathrm{m/s^2} \Rightarrow$ $N = (50\,\mathrm{kg})(10 - 2\,\mathrm{m/s^2}) = 400\,\mathrm{N}.$

(c) $a_y = 0 \Rightarrow$ $N = 500\,\mathrm{N}$ as in (a).

(d)

$$a_y = \frac{v_{fy} - v_{iy}}{t_f - t_i} = \frac{0 - (-6.0\,\mathrm{m/s})}{4.0\,\mathrm{s}} = +1.5\,\mathrm{m/s^2}$$

$$N = (50\,\mathrm{kg})(10 + 1.5\,\mathrm{m/s^2}) = 575\,\mathrm{N}.$$

4. A meter stick moves past you at high speed. The stick's motion relative to you is parallel to its long axis. The back of the stick passes you 1.30 nanoseconds (1.30×10^{-9} s) after the front of the stick passes. How fast is the stick moving in your rest frame?

Let $L_0 = 1\text{m}$ be the proper length of the meter stick in its own rest frame.

You observe the Lorentz-contracted length

$$L = \frac{L_0}{\gamma_v}.$$

Accordingly, the time for the stick to pass you is

$$T = \frac{L}{v} = \frac{L_0}{\gamma_v v}.$$

Need to solve for v:

$$\frac{v}{c} cT = \frac{L_0}{\gamma_v}$$

$$\left(\frac{v}{c}\right)^2 (cT)^2 = \left[1 - \left(\frac{v}{c}\right)^2\right] L_0^2$$

$$\left[L_0^2 + (cT)^2\right] \left(\frac{v}{c}\right)^2 = L_0^2$$

$$\left(\frac{v}{c}\right)^2 = \frac{L_0^2}{L_0^2 + (cT)^2} = \frac{1}{1 + (cT/L_0)^2}$$

$$v = \frac{c}{\sqrt{1 + (cT/L_0)^2}}$$

Here, $\frac{cT}{L_0} = \frac{(3.0 \times 10^8 \text{m/s})(1.30 \times 10^{-9} \text{s})}{1\text{m}} = 0.39$

$\Rightarrow$ $v = 0.93c = 2.8 \times 10^8 \text{m/s}.$

5. A bungee jumper steps off a high bridge and simultaneously shouts out in excitement. After she has been in free-fall for 1.2 seconds, she hears the echo of her shout reflected from the ground directly below the bridge. Assume that gravity is the only force acting on the jumper during these 1.2 seconds. Take the speed of sound to be 340 m/s.

   (a) How far has the jumper fallen when she hears the echo?

   (b) How high above the ground is the bridge?

Let us measure positions upwards from ground level.

(a) The position of the jumper is

$$y_j = y_0 - \tfrac{1}{2} g t^2$$

The distance fallen is

$$\begin{aligned} y_0 - y_j &= \tfrac{1}{2} g t^2 \\ &= \tfrac{1}{2}(10\,\text{m/s}^2)(1.2\,\text{s})^2 \\ &= 7.2\,\text{m}. \end{aligned}$$

(b) After the same time $t = 1.2\,\text{s}$, the sound has traveled all the way from the bridge to the ground and back to height y_j:

$$\begin{aligned} v_s t &= y_0 + y_j \\ &= 2y_0 - \tfrac{1}{2} g t^2 \\ \Rightarrow \quad y_0 &= \tfrac{1}{2} v_s t + \tfrac{1}{4} g t^2 \\ &= \tfrac{1}{2}(340\,\text{m/s})(1.2\,\text{s}) \\ &\quad + \tfrac{1}{4}(10\,\text{m/s}^2)(1.2t)^2 \\ &= 208\,\text{m}. \end{aligned}$$

6. A ball of mass m travels in a vertical circle on the end of a massless string of length L. Because of gravity, the ball's speed changes as it goes round the circle. Write down algebraic expressions for the magnitude of (i) the ball's acceleration, and (ii) the tension in the string, in each of the following cases:

(a) The ball is at the top of the circle, and has speed v_t.

(b) The ball is at the bottom of the circle, and has speed v_b.

(c) The ball is at the same height as the center of the circle, and has speed v_m.

For an object in circular motion,

$$\sum_i \vec{F}_i = m\vec{a}$$

where

$$\vec{a} = -\frac{v^2}{r}\hat{r} + \frac{dv}{dt}\hat{t}.$$

(a) At the top, N2 gives

$$x: \quad 0 = ma_x$$

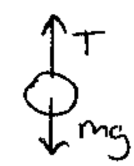
$$y: \quad -T - mg = ma_y = -\frac{mv_t^2}{L}$$

$$\Rightarrow \quad |\vec{a}| = \frac{v_t^2}{L}, \quad T = m\left(\frac{v_t^2}{L} - g\right).$$

(b) At the bottom, N2 gives

$$x: \quad 0 = ma_x$$

$$y: \quad T - mg = ma_y = +\frac{mv_b^2}{L}$$

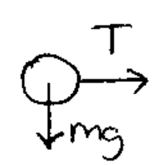


$$\Rightarrow \quad |\vec{a}| = \frac{v_b^2}{L}, \quad T = m\left(\frac{v_b^2}{L} + g\right).$$

(c) At the mid-point, N2 gives

$$x: \quad T = ma_x = \frac{mv_m^2}{L}$$

$$y: \quad -mg = ma_y$$



$$\Rightarrow \quad |\vec{a}| = \sqrt{\left(\frac{v_m^2}{L}\right)^2 + g^2}, \quad T = \frac{mv_m^2}{L}$$

7. An intergalactic speed checkpoint observes a rocket joyrider whizzing by at $0.60c$ in a $0.50c$ zone. A patrolman jumps into his cruiser and, after a 5.0-minute delay for take-off and acceleration maneuvers, sets off in pursuit at a speed of $0.70c$ (both the delay and the speed being as measured at the checkpoint).

(a) What velocity does the patrolman observe the joyrider to have?

(b) How long after he passes the checkpoint does the patrolman overtake the joyrider, as measured on the clock in his space cruiser?

Let S be the rest frame of the checkpoint

S' " " " " " " patrolman moving at $v = 0.7c$ in S

$V_x = 0.6c$ be the velocity of the joyrider in S

$T = 5$ min be the joyrider's lead time in S.

(a) By the Lorentz velocity transformation, the patrolman observes the joyrider's velocity to be

$$V_x' = \frac{V_x - v}{1 - vV_x/c^2} = \frac{0.60c - 0.70c}{1-(0.60)(0.70)} = -0.17c$$

The patrolman observes the joyrider moving along the $-x'$ direction at $0.17c$.

(b) In frame S, the patrolman's velocity relative to the joyrider is $v - V_x$.

$\Rightarrow$ The patrolman makes up the joyrider's lead distance $V_x T$ in a time

$$\Delta t = \frac{V_x T}{v - V_x} = 30 \text{ min}.$$

Since moving clocks run slow, the time that elapses in frame S' is

$$\Delta t' = \frac{\Delta t}{\gamma_v}$$
$$= \sqrt{1-(0.70)^2}\,(30 \text{ min})$$
$$= 21.4 \text{ min} = 1290 \text{ s}$$

8. Explorer Ed, standing in a flat desert region on Earth, observes two flashes of light: an orange flash at a point 25 km west of him and a blue flash at a point 20 km east of him. Ed also sees Astronaut Alice in a spaceship traveling due east at a speed of $0.95c$. In Alice's rest frame, the blue flash occurs 0.2 milliseconds (0.2×10^{-3} s) before the orange flash. For the purposes of this question, assume that Earth can be treated as an inertial reference frame.

   (a) What is the time interval between the flashes in Ed's rest frame? Which flash occurs first in Ed's frame?

   (b) Is it possible to find an inertial frame in which the two flashes occur at the same spatial location? If so, what is the velocity of that frame relative to Ed's rest frame?

   (c) Is it possible to find an inertial frame in which the two flashes occur at the same time? If so, what is the velocity of that frame relative to Ed's rest frame?

Let S be Ed's rest frame and S' be Alice's rest frame, moving at $v = 0.95c$ in S.

$$\Rightarrow \quad \Delta x = x_{blue} - x_{orange} = (20\,\text{km}) - (-25\,\text{km})$$
$$= 45\,\text{km}$$
$$\Delta t' = t'_{blue} - t'_{red}$$
$$= -0.2\,\text{ms} \qquad \text{(note the sign)}$$

(a) The Lorentz transformation gives

$$\Delta t' = \gamma_v (\Delta t - v\Delta x / c^2)$$
$$\Rightarrow \quad \Delta t = \frac{\Delta t'}{\gamma_v} + \frac{v\Delta x}{c^2}$$
$$= \frac{-2.0\times10^{-4}\,\text{s}}{3.20} + 0.95\,\frac{45\times10^{3}\,\text{m}}{3.0\times10^{8}\,\text{m/s}}$$
$$= +8.0\times10^{-5}\,\text{s}$$

In Ed's frame the orange flash occurs first.

(b) Look for $0 = \Delta x' = \gamma_v (\Delta x - v\Delta t)$

$$\frac{v}{c} = \frac{\Delta x}{c\Delta t} = \frac{45\times10^{3}\,\text{m}}{(3.0\times10^{8}\,\text{m/s})(8.0\times10^{-5}\,\text{s})}$$
$$= 1.88 > 1 \quad \Rightarrow \underline{\text{not}} \text{ possible}$$

(c) Look for $0 = \Delta t' = \gamma_v (\Delta t - v\Delta x / c^2)$

$$\frac{v}{c} = \frac{c\Delta t}{\Delta x}$$
$$= \frac{1}{1.88} = 0.53 < 1 \quad \Rightarrow \underline{\text{is}} \text{ possible}$$

9. A passenger train is traveling at 45 m/s when the engineer sees that a freight train is only 1.5 km ahead on the same track. The freight train is moving at a constant speed of 10 m/s in the same direction as the passenger train. The engineer immediately applies the passenger train's brakes. What is the minimum magnitude of the resulting acceleration (in m/s^2) if a collision is just to be avoided?

Hint: Consider the freight train's position relative to the passenger train.

There are many approaches to solving this problem.

Following the hint, let us work in the frame of the passenger train. Then the freight train obeys

$$a > 0$$
$$v = v_0 + at$$
$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

where

$$v_0 = 10\,\text{m/s} - 45\,\text{m/s} = -35\,\text{m/s}$$
$$x_0 = 1500\,\text{m}.$$

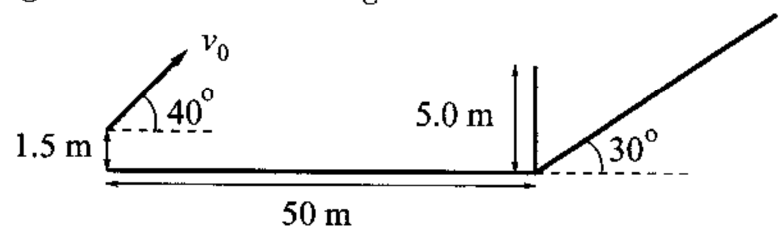
A collision will be avoided provided that the velocity becomes non-negative before x becomes negative.

$$v^2 = v_0^2 + 2a(x - x_0).$$

Look for $\qquad 0 \leq v_0^2 - 2ax_0 \quad$ (since want $x \geq 0$)

$$\Rightarrow \qquad a \geq \frac{v_0^2}{2x_0} = \frac{(-35\,\text{m/s})^2}{2\,(1500\,\text{m})} = 0.41\,\text{m/s}^2$$

10. A child throws a stone at an initial speed v_0 and at an initial angle of 40° above the level ground. The stone is released at a height of 1.5 m above the ground, and travels towards a 5.0-m-high fence that is located 50 m away horizontally from the launch point. A slope rises at an angle of 30° immediately behind the fence, as shown in the diagram. Neglect all effects of air drag.



(a) Suppose that $v_0 = 23\,\text{m/s}$. Calculate the position (x and y coordinates measured from ground level directly under the stone's point of release) where the stone first makes contact with the ground, the fence, or the slope.

(b) Repeat part (a) for the case where $v_0 = 25\,\text{m/s}$.

The stone's trajectory is $y = y_0 + x\tan\phi_0 - \frac{g}{2}\left(\frac{x}{V_0\cos\phi_0}\right)^2$ (1)

(a) Look at $x = x_f = 50$ m to see whether the stone falls short of / hits / passes over the fence:

$$y = (1.5\text{m}) + (50\text{m})\tan 40° - \tfrac{1}{2}(10\,\text{m/s}^2)\left[\frac{50\text{m}}{(23\text{m/s})\cos 40°}\right]^2$$
$$= 3.2\text{m}$$

$\Rightarrow$ Stone hits the fence at $x = 50$m, $y = 3.2$m.

(b) Again look at $x = x_f$:

$$y = (1.5\text{m}) + (50\text{m})\tan 40° - \tfrac{1}{2}(10\,\text{m/s}^2)\left[\frac{50\text{m}}{(25\text{m/s})\cos 40°}\right]^2$$
$$= 9.4\text{m}$$

$\Rightarrow$ Stone passes over the fence and hits the slope, which rises at angle $\theta = 30°$ from $x = x_f$, ie,

$$y = (x - x_f)\tan\theta \quad (2)$$

The landing point is the intersection of (1) and (2), where

$$\frac{g}{2V_0^2\cos^2\phi_0}x^2 - (\tan\theta - \tan\phi_0)x - (y_0 + x_f\tan\theta) = 0$$
$$0.01363\,x^2 \quad - 0.2617\,x \quad - 30.368 \quad = 0$$

$\Rightarrow \quad x = 57.8\text{m}, \qquad y = 4.5\text{m}$