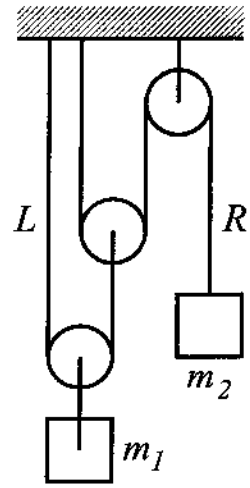
PHY 2060 SPRING 2008 - EXAM 2 SOLUTION

1. Two masses, m_1 and m_2, are suspended from the ceiling using two ideal strings ("L" for left, and "R" for right) and three massless, frictionless pulleys. Consider the motion during a short period after the system is released from rest in the configuration shown in the diagram. Place a check to the left of any/all of the following statements that **must** be true:



× i. The tension in R will be twice the tension in L.

× ii. The system will remain at rest provided that $m_1 = 2m_2$.

× iii. The distance moved by m_1 will be four times the distance moved by m_2.

× iv. The tension in L will be $2m_1 g$.

✓ v. The distance moved by m_2 will be four times the distance moved by m_1.

(i) Applying N2 to the middle pulley,

$$2T_R - T_L = m_{pulley}\, a_{pulley} = 0. \qquad \Rightarrow \text{FALSE}$$

(ii) In equilibrium, N2 gives

$$\begin{aligned} 2T_L - m_1 g &= 0 && \text{(left pulley)} \\ 2T_R - T_L &= 0 && \text{(middle pulley)} \\ T_R - m_2 g &= 0 && \text{(mass } m_2\text{)} \end{aligned}$$

$$\Rightarrow \qquad m_1 = 4m_2. \qquad \Rightarrow \text{FALSE}$$

(iii) The middle pulley will move twice as far as m_1, and m_2 will move twice as far as the middle pulley, so m_2 will move four times as far as m_1. $\Rightarrow$ FALSE

(iv) In a general (non-equilibrium) case, N2 gives

$$\begin{aligned} T_1 - m_1 g &= m_1 a_1 && \text{(mass } m_1\text{)} \\ 2T_L - T_1 &= 0 && \text{(left pulley)} \end{aligned}$$

$$\Rightarrow \qquad T_L = \tfrac{1}{2} m_1 (g + a_1) \qquad \Rightarrow \text{FALSE}$$

(v) See (iii) above. $\Rightarrow$ TRUE

2. A car of mass 800 kg is traveling at velocity 20 m/s in the $+x$ direction when it undergoes a totally inelastic collision with a second car of mass 1,000 kg moving at 30 m/s. Find the cars' speed immediately after the collision, if before the collision the second car was moving along

   (a) the $-x$ direction;

   (b) the $+y$ direction.

In a totally inelastic collision ($\vec{v}_{2f} = \vec{v}_{1f}$), momentum conservation gives

$$(m_1 + m_2)\vec{v}_f = m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}$$

$$\vec{v}_f = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{m_1 + m_2}$$

(a)

$$v_{fx} = \frac{(800\,kg)(20\,m/s) + (1000\,kg)(-30\,m/s)}{800 + 1000\,kg}$$

$$= -7.78\,m/s$$

$$v_{fy} = 0$$

speed

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$

$$= 7.8\,m/s$$

(b)

$$v_{fx} = \frac{(800\,kg)(20\,m/s) + (1000\,kg)(0\,m/s)}{800 + 1000\,kg}$$

$$= 8.89\,m/s$$

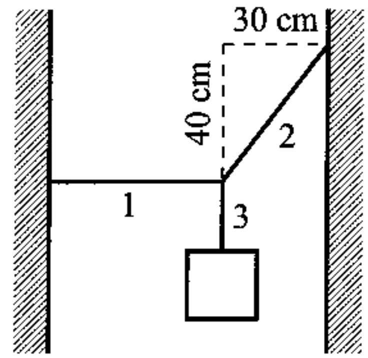
$$v_{fy} = \frac{(800\,kg)(0\,m/s) + (1000\,kg)(30\,m/s)}{800 + 1000\,kg}$$

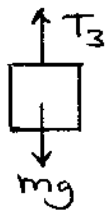
$$= 16.67\,m/s$$

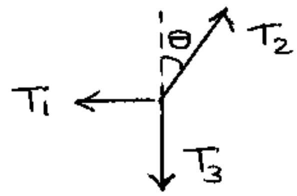
$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$

$$= 18.9\,m/s$$

3. A 3.0-kg mass is suspended at rest using three ideal strings (labeled 1, 2, and 3 in the diagram) that meet at a knot. The other ends of strings 1 and 2 are attached to two rigid walls. Calculate the tensions T_1, T_2, and T_3 in the three strings.







N2 applied to the mass gives

y: $\quad T_3 - mg = 0 \quad$ or $\quad T_3 = mg$

N2 applied to the knot gives

x: $\quad T_2 \sin\theta - T_1 = 0$

y: $\quad T_2 \cos\theta - T_3 = 0 \quad$ or $\quad T_2 = mg\sec\theta$

$T_1 = mg\tan\theta$

Substituting $m = 3.0$ kg and $\theta = \tan^{-1}\frac{30}{40}$,

$T_1 = 22.5$ N

$T_2 = 37.5$ N

$T_3 = 30$ N

4. A bullet of mass 60 g is fired from a gun with an initial velocity of 150 m/s at an angle of 50° above the horizontal ground. During its flight, the bullet explodes into two fragments: one of mass 10 g, the other of mass 50 g. The internal forces acting on the fragments during the explosion are directed horizontally and within the vertical plane that contains the bullet's trajectory before the explosion. The lighter fragment eventually hits the ground 3.7 km from the gun. How far from the gun does the heavier fragment land? Neglect air resistance.

Since all internal forces are horizontal, the fragments have the same vertical velocity component after the collision as they did before, and both land at the same moment.

Their center of mass lands a distance from the gun

$$R = \frac{V_0^2}{g} \sin 2\phi_0$$

$$= \frac{(150\,m/s)^2}{10\,m/s^2} \sin(2 \times 50°) = 2.22\,km.$$

Since the cm of two point masses is

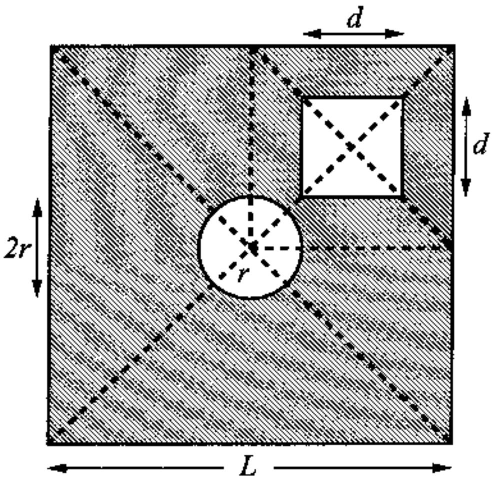
$$R = \frac{m_1 R_1 + m_2 R_2}{m_1 + m_2}$$

$$\Rightarrow \qquad R_2 = \frac{1}{m_2}\left[(m_1+m_2)R - m_1 R_1\right]$$

$$= \frac{1}{50g}\left[(10g+50g)(2.22\,km) - (10g)(3.7km)\right]$$

$$= 1.9\,kg$$

5. A plate of uniform thickness t and uniform density ρ has the shape of a a square of sides L containing two holes: one a circle of radius r centered on the middle of the square outer perimeter; the other a small square of sides d, centered three-quarters of the way along the diagonal leading from the plate's bottom left corner to its top right corner. (See the diagram.)



Calculate the distance of the plate's center of mass from its bottom left corner.

Let's measure all distances along the diagonal from the bottom-left corner.

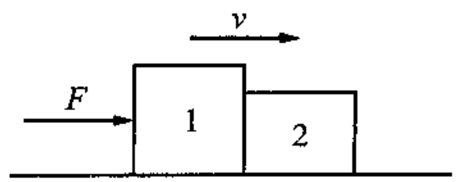
Using the "negative-mass" method

$$r_{cm} = \frac{m_o r_{cm,o} - m_{h1} r_{cm,h1} - m_{h2} r_{cm,h2}}{\underbrace{m_o}_{\text{mass of plate with no holes}} - m_{h1} - \underbrace{m_{h2}}_{\text{mass of plate that would fill hole 2}}}$$

$$= \frac{L^2 \frac{L}{\sqrt{2}} - \pi r^2 \frac{L}{\sqrt{2}} - d^2 \frac{3}{2} \frac{L}{\sqrt{2}}}{L^2 - \pi r^2 - d^2}$$

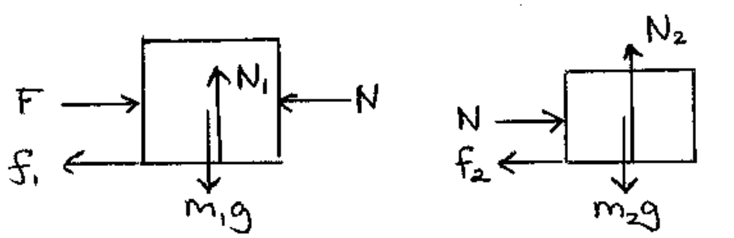
$$= \frac{L^2 - \pi r^2 - \frac{3}{2} d^2}{L^2 - \pi r^2 - d^2} \frac{L}{\sqrt{2}}$$

6. Two blocks ("1" and "2") are pushed in a straight line at a constant speed v across a horizontal surface. This is accomplished by applying an external force of magnitude F to block 1, while block 2 is pushed along in front of block 1, as shown in the diagram. Block j ($j = 1$ or 2) has mass m_j and its coefficient of kinetic friction with the surface is μ_j. Assume that $\mu_2 < \mu_1$.



(a) What is the magnitude F of the applied force that sustains this motion?

(b) The external force is now removed. Once the blocks have come to rest, how far apart will they be? (Remember that the blocks are in contact, i.e., at zero separation, during the whole time the force F is being applied.)



(a) Treating the two blocks as a single rigid body

$$F - f_1 - f_2 = 0$$

Since $\quad f_j = \mu_j N_j$

and $\quad N_j = m_j g$

$\Rightarrow \quad F = (\mu_1 m_1 + \mu_2 m_2) g$

(b) The blocks now separate and each obeys

$$-f_j = m_j a_j$$

or $\quad a_j = -\mu_j g$

The distance traveled before coming to rest is given by

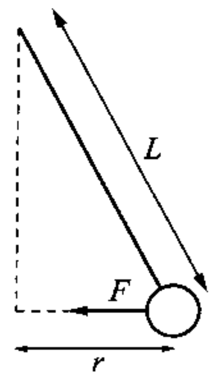
$$\overset{0}{\cancel{v_f^2}} = v_i^2 + 2a_j x_j$$

$$x_j = -\frac{v^2}{2a_j} = \frac{v^2}{2\mu_j g}$$

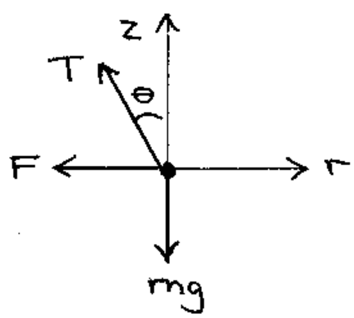
Thus, the final separation of the blocks is

$$x_2 - x_1 = \frac{v^2}{2g}\left(\frac{1}{\mu_2} - \frac{1}{\mu_1}\right)$$

7. A variant of the conical pendulum consists of a small ball of mass m suspended on the end of an ideal string of length L. The upper end of the string is attached to a fixed support, and the pendulum is set in motion so that the ball travels in a horizontal circle of radius r at a uniform speed v. (The diagram shows a side view, where the instantaneous velocity of the ball is into the page.) Unlike the conventional conical pendulum, the ball experiences a third force in addition to its weight and the string tension, namely, a horizontal force of magnitude $F = kr$, where k is a constant. This force always points towards the center of the ball's circular path.



Find an expression for the speed v in terms of other quantities specified in the problem and g, the acceleration due to gravity. Neglect air resistance.



N2 along z: $T\cos\theta - mg = 0$ or $T = mg\sec\theta$

" " r: $-T\sin\theta - F = -\frac{mv^2}{r}$

$$v^2 = \frac{r}{m}(T\sin\theta + F)$$

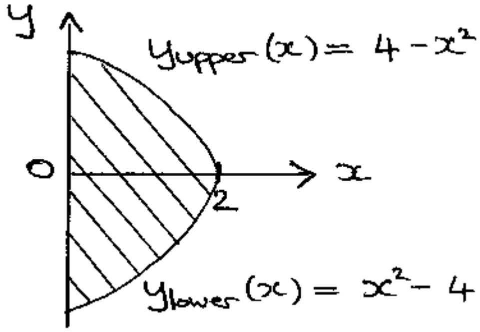
$$= \frac{r}{m}(mg\tan\theta + kr)$$

But $\sin\theta = \frac{r}{L}$

$$\tan\theta = \frac{r}{\sqrt{L^2 - r^2}}$$

$\Rightarrow$ $v = \sqrt{\frac{gr^2}{\sqrt{L^2-r^2}} + \frac{kr^2}{m}}$

8. A plate of uniform thickness t and uniform density ρ has a shape in the xy plane satisfying $x > 0$, $y < 4 - x^2$, and $y > x^2 - 4$. Calculate the x and y components of the plate's center of mass position.



Since $y_{lower}(x) = -y_{upper}(x)$, symmetry dictates that

$$y_{cm} = 0.$$

We need to calculate

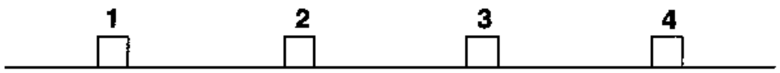
$$x_{cm} = \frac{\int x\,dA}{\int dA}$$

$$\begin{aligned}
\int x\,dA &= \int_0^2 x\left[y_{upper}(x) - y_{lower}(x)\right] dx \\
&= \int_0^2 x\,(4 - x^2 - x^2 + 4)\,dx \\
&= \int_0^2 (8x - 2x^3)\,dx \\
&= \left[4x^2 - \tfrac{1}{2}x^4\right]_0^2 \\
&= 8
\end{aligned}$$

$$\begin{aligned}
\int dA &= \int_0^2 \left[y_{upper}(x) - y_{lower}(x)\right] dx \\
&= \int_0^2 (8 - 2x^2)\,dx \\
&= \left[8x - \tfrac{2}{3}x^3\right]_0^2 \\
&= \frac{32}{3}
\end{aligned}$$

$$\Rightarrow \quad x_{cm} = \frac{8}{32/3} = \frac{3}{4}$$

9. Four carts are spaced out as shown in the diagram along a straight track that permits the carts to move (without friction) in only one dimension. The carts have masses $m_2 = m_4 = m$ and $m_1 = m_3 = 3m$. Carts 1, 3, and 4 are initially stationary, while cart 2 is initially moving towards cart 3 at speed v. Find the final velocity of each cart, assuming that collisions involving cart 1 and/or cart 4 are totally inelastic, while collisions involving only carts 2 and 3 are elastic.



Collision 1 between carts 2 and 3:

Elastic $\Rightarrow$

$$V_{2f} = \frac{m_2 - m_3}{m_2 + m_3} V_{2i} + \frac{2m_3}{m_2 + m_3} V_{3i}$$
$$= \frac{m - 3m}{m + 3m} V + \frac{2 \cdot 3m}{m + 3m} 0$$
$$= -\frac{1}{2} V$$

$$V_{3f} = \frac{m_3 - m_2}{m_3 + m_2} V_{3i} + \frac{2m_2}{m_3 + m_2} V_{2i}$$
$$= \frac{3m - m}{3m + m} 0 + \frac{2m}{3m + m} V$$
$$= \frac{1}{2} V$$

Collision 2 between carts 1 and 2:

Totally inelastic $\Rightarrow$ $V_{1f} = V_{2f} = \dfrac{m_1 V_{1i} + m_2 V_{2i}}{m_1 + m_2}$

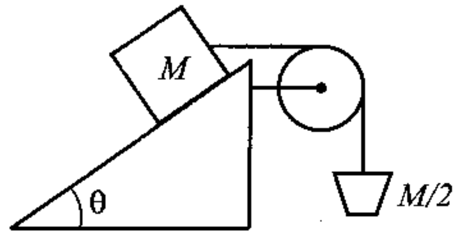
$$= \frac{3m \cdot 0 + m\left(-\frac{1}{2}V\right)}{3m + m}$$
$$= -\frac{1}{8} V$$

Collision 3 between carts 3 and 4:

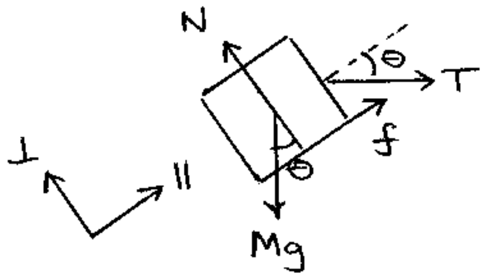
Totally inelastic $\Rightarrow$ $V_{3f} = V_{4f} = \dfrac{m_3 V_{3i} + m_4 V_{4i}}{m_3 + m_4}$

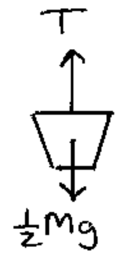
$$= \frac{3m \cdot \frac{1}{2}V + m \cdot 0}{3m + m}$$
$$= \frac{3}{8} V$$

10. A block of mass M rests on an plane inclined at angle θ to the horizontal. The coefficient of static friction between the block and the slope is 0.3. An ideal string runs horizontally from the block and passes over a massless, frictionless pulley. A bucket, of mass $M/2$, is suspended from the other end of the string, as shown in the diagram.



Calculate the range of angles θ for which this system will remain in static equilibrium.





In equilibrium, N2 for the bucket gives

$$T - \tfrac{1}{2}Mg = 0 \quad \text{or} \quad T = \tfrac{1}{2}Mg$$

and N2 for the block gives

$$\perp: \quad N - Mg\cos\theta - T\sin\theta = 0$$

$$\Rightarrow N = Mg\cos\theta + T\sin\theta$$

$$= Mg(\cos\theta + \tfrac{1}{2}\sin\theta)$$

$$\parallel: \quad T\cos\theta + f - Mg\sin\theta = 0$$

$$\Rightarrow f = Mg\sin\theta - T\cos\theta$$

$$= Mg(\sin\theta - \tfrac{1}{2}\cos\theta)$$

We know that $|f| \leq \mu_s N$

$$|\sin\theta - \tfrac{1}{2}\cos\theta| \leq \mu_s(\cos\theta + \tfrac{1}{2}\sin\theta),$$

i.e.,

$$\sin\theta - \tfrac{1}{2}\cos\theta \leq \mu_s(\cos\theta + \tfrac{1}{2}\sin\theta)$$

$$(1 - \mu_s/2)\tan\theta \leq \tfrac{1}{2} + \mu_s \quad \text{or} \quad \theta \leq \tan^{-1}\frac{1+2\mu_s}{2-\mu_s}$$

<u>and</u>

$$\tfrac{1}{2}\cos\theta - \sin\theta \leq \mu_s(\cos\theta + \tfrac{1}{2}\sin\theta)$$

$$-(1 + \mu_s/2)\tan\theta \leq -(\tfrac{1}{2} - \mu_s) \quad \text{or} \quad \theta \geq \tan^{-1}\frac{1-2\mu_s}{2+\mu_s}$$

For $\mu_s = 0.3$, these conditions mean $9.9^\circ \leq \theta \leq 43.3^\circ$