PHY 2060 Spring 2008 - Exam 3 Solution

1. Two masses undergo a non-relativistic collision described by a coefficient of restitution ε. The collision is studied by two observers in different inertial frames of reference. Observer C is in the center-of-mass frame of the colliding masses, and measures the total kinetic energy of the two masses to change from $K_{C,i}$ immediately before the collision to $K_{C,f}$ immediately after. Observer L is in the laboratory frame, and measures the total kinetic energy to change from $K_{L,i}$ to $K_{L,f}$.

   Place a check to the left of any/all of the following statements that **must** be true:

   ✓ (a) $K_{C,f} = \varepsilon^2 K_{C,i}$.

   ✓ (b) $K_{L,f} - K_{L,i} = K_{C,f} - K_{C,i}$.

   ✓ (c) $K_{C,i} \leq K_{L,i}$.

   × (d) $K_{L,f} = \varepsilon^2 K_{L,i}$.

   × (e) $K_{C,f} \geq K_{L,f}$.

(a) is a fundamental property of two-body collisions.

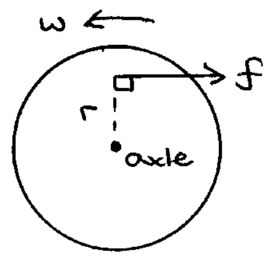
(b) and (c) follow from $K_L = K_C + \frac{1}{2}\left(\sum_j m_j\right) V_{cm,L}^2$.

(d) and (e) are inconsistent with these facts.

2. A uniform sphere of mass M and radius R is initially spinning freely at angular velocity ω_0 about a light, thin axle passing through the sphere's center. A needle tip is pressed against the sphere's surface at a point a distance r from the axis of rotation. The needle tip exerts a constant frictional force of magnitude f on the sphere until the sphere comes to a halt.

   (a) Find the magnitude of the sphere's angular acceleration while it is coming to a halt.

   (b) Through how many revolutions does the sphere rotate after the needle tip is first pressed against it?



(a)

$$\Sigma \tau = I\alpha$$
$$-fr = \frac{2}{5}MR^2\alpha$$
$$|\alpha| = \frac{5fr}{2MR^2}$$

(b) Since α is constant,

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\phi$$
$$\Delta\phi = \frac{\omega^2 - \omega_0^2}{2\alpha}$$

For $\omega = 0$,

$$\Delta\phi = \frac{\omega_0^2}{2|\alpha|} = \frac{\omega_0^2 MR^2}{5fr}$$

This angle is naturally expressed in radians.

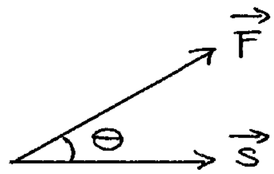
$$\#\text{ revolutions} = \frac{\Delta\phi}{2\pi} = \frac{\omega_0^2 MR^2}{10\pi fr}$$

3. A sled of mass 30 kg initially sits at rest on horizontal snow. A girl sets the sled in motion by pulling with a constant force of 100 N on a rope oriented at 40° above the horizontal. Neglect friction between the sled and the snow.

   (a) How much work has the girl performed on the sled by the time the sled has traveled 5.0 m?

   (b) What is the sled's speed at the moment it is 5.0 m from its start point?

(a)



Work $W = Fs\cos\theta$

$= (100\,N)(5.0\,m)\cos 40°$

$= 380\,N$

(b) Work-energy theorem:

$$\Delta K = W$$

$$\frac{1}{2}m(v_f^2 - v_i^2) = W$$

Here, $v_i = 0$.

$\Rightarrow$ $$v_f = \sqrt{\frac{2W}{m}}$$

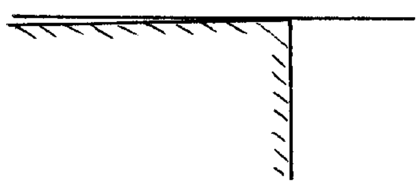
$$= 5.1\,m/s$$

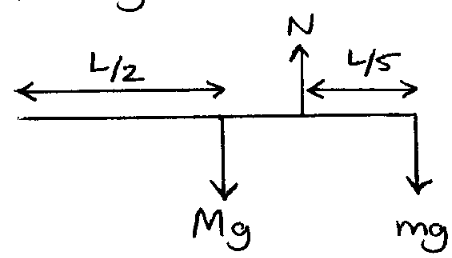
4. A uniform plank of mass M and length L rests with four-fifths of its length on horizontal ground and the remainder (at one end) extending over the edge of a cliff. A person of mass m wishes to walk to the unsupported end of the plank.

(a) What is the maximum value of m that will allow the person to reach the unsupported end of the plank without it tipping?

(b) Assuming that m is too big to allow the end to be reached safely, how close to the unsupported end of the plank can the person get without the plank tipping?

(a) Consider a moment when the plank is just about to tip, so the normal force exerted by the ground on the plank acts only at the cliff edge:



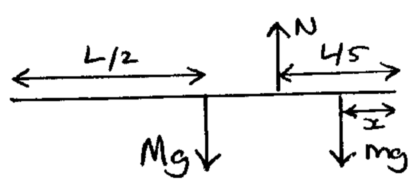


Applying $\Sigma\tau = 0$ about the cliff edge in this limiting case,

$$Mg\left(L - \frac{L}{2} - \frac{L}{5}\right) - mg\,\frac{L}{5} = 0$$

$$\Rightarrow \quad m = \frac{3}{2}M$$

(b)



x = distance from end of plank

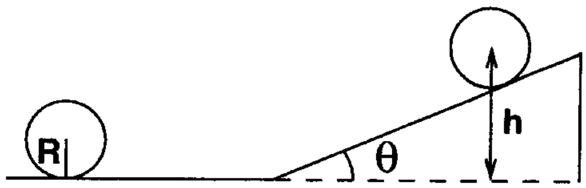
Again applying $\Sigma\tau = 0$ about the cliff edge

$$Mg\left(L - \frac{L}{2} - \frac{L}{5}\right) - mg\left(\frac{L}{5} - x\right) = 0$$

Closest approach to edge: $$x = \frac{L}{5} - \frac{3ML}{10m}$$

$$= \left(2 - \frac{3M}{m}\right)\frac{L}{10}$$

5. A uniform sphere of mass M and radius R rolls without slipping at a constant center-of-mass speed v across a horizontal surface until it comes to a slope inclined at angle θ to the horizontal. The sphere travels up the slope until its center of mass reaches a maximum height h above the horizontal surface.



(a) Find h, assuming that the sphere rolls without slipping on the slope.

(b) Find h, assuming instead that the slope is frictionless.

(a) Work-energy theorem: $\Delta K = W$

Here

$$K_i = \frac{1}{2}Mv^2 + \frac{1}{2}I_{cm}\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\,\frac{2}{5}MR^2\underbrace{\left(\frac{v}{R}\right)^2}_{\text{no slip}}$$

$$= \frac{7}{10}Mv^2$$

$$K_f = 0$$

$$W = -Mg\underbrace{(h-R)}_{\Delta y \text{ for cm of sphere}}$$

$$\Rightarrow \quad -\frac{7}{10}Mv^2 = -Mg(h-R)$$

$$h = R + \frac{7v^2}{g}$$

(b) The only difference from (a) is that the sphere's angular velocity remains equal to v/R at the end

$$\Rightarrow \quad K_f = \frac{1}{2}\,\frac{2}{5}MR^2(v/R)^2$$

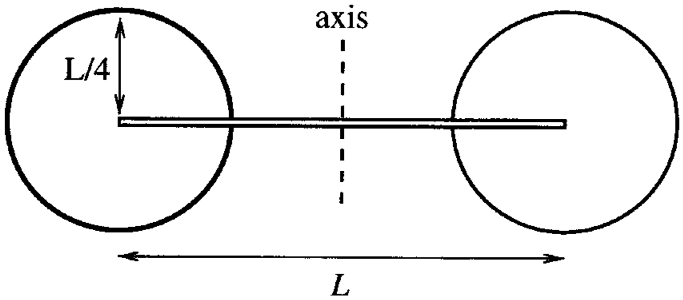
$$= \frac{1}{5}Mv^2$$

So

$$-\frac{1}{2}Mv^2 = -Mg(h-R)$$

$$h = R + \frac{v^2}{2g}$$

6. A space station can be thought of as consisting of two uniform spherical shells, each of mass M and radius $L/4$, centered on either end of an elevator shaft, also of mass M. The shaft is of length L, has negligible dimensions perpendicular to its length, and its mass is distributed uniformly along its length. What is the rotational inertia of the space station when it rotates about an axis halfway between the spheres, perpendicular to the elevator shaft?



$$I_{total} = I_{shaft} + I_{left\ shell} + I_{right\ shell}$$

$$I_{shaft} = \frac{1}{12} mL^2$$

$$I_{left\ shell} = I_{right\ shell}$$

$$= I_{cm} + Md^2 \quad \text{(parallel axis thm)}$$

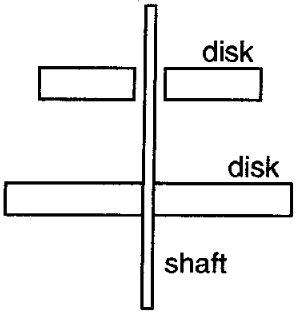
$$= \frac{2}{3} m\left(\frac{L}{4}\right)^2 + m\left(\frac{L}{2}\right)^2$$

$$= \frac{7}{24} mL^2$$

$$I_{total} = \left(\frac{1}{12} + 2 \times \frac{7}{24}\right) mL^2$$

$$= \frac{2}{3} mL^2$$

7. The diagram shows two cylindrical disks, each of uniform thickness and density. The lower disk has mass 360 g and radius 5.5 cm. It is initially rotating at 80 rev/min on a light, frictionless shaft of negligible radius. The upper disk, of mass 250 g and radius 5.0 cm, is initially at rest. It is then allowed to drop freely down the shaft onto the lower disk, and the two disks exert frictional forces on each other until, 7.0 sec after they first make contact, the disks reach a common angular velocity.



(a) What is the common angular velocity of the two disks?

(b) What is the magnitude of the constant torque (measured about the shaft) exerted on the upper disk during the time that it is accelerating to reach the same angular velocity as the lower disk?

(a) By conservation of angular momentum about the shaft

$$(I_U + I_L)\,\omega_f = I_U \cancel{\omega_U}^{\,0} + I_L \omega_L$$

$$\omega_f = \frac{I_L \omega_L}{I_U + I_L}$$

$$= \frac{\frac{1}{2} m_L R_L^2}{\frac{1}{2} m_U R_U^2 + \frac{1}{2} m_L R_L^2}\,\omega_L$$

$$= 51 \text{ rev/min} \quad \text{or} \quad 5.3 \text{ rad/s}$$

(b)

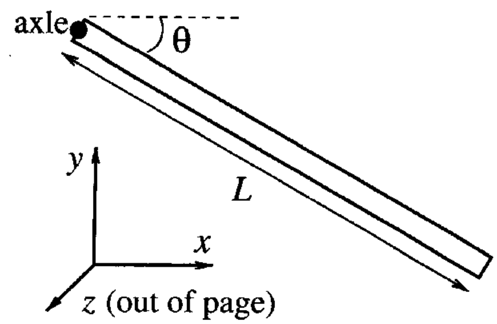
$$\tau = I\alpha$$

$$= \frac{1}{2} m_U R_U^2 \frac{\omega_f - \cancel{\omega_0}^{\,0}}{\Delta t}$$

$$= \frac{1}{2}(250\text{g})(5.0\text{cm})^2 \frac{5.3 \text{ rad/s}}{7.0 \text{ s}} \times \left(\frac{1\text{kg}}{1000\text{g}}\right) \times \left(\frac{1\text{m}}{100\text{cm}}\right)^2$$

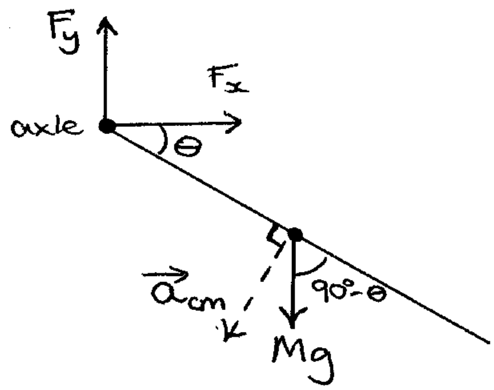
$$= 2.4 \times 10^{-4} \text{ Nm}$$

8. A thin, uniform rod of mass M and length L is attached at one end to a horizontal axle, so that the rod can rotate freely in a horizontal plane. The rod is held in the position shown in the figure, making an initial angle θ to the horizontal, and is then released.



(a) What are the x, y, and z components of the rod's angular acceleration immediately after the rod is released?

(b) What are the x, y, and z components of the force exerted by the axle on the rod immediately after the rod is released?



(a) Taking torques about the axle

$$I\vec{\alpha} = \Sigma\vec{\tau}$$

Clearly $\alpha_x = \alpha_y = 0$

Along z $\frac{1}{3}ML^2\,\alpha_z = -Mg\,\frac{L}{2}\sin(90°-\theta)$

$$\alpha_z = -\frac{3g\cos\theta}{2L}$$

(b) $\Sigma\vec{F} = M\vec{a}_{cm}$

By kinematics, $\vec{a}_{cm} = \frac{L}{2}|\vec{\alpha}|\,(-\sin\theta, -\cos\theta, 0)$

Along x, $F_x = M a_{cm,x}$

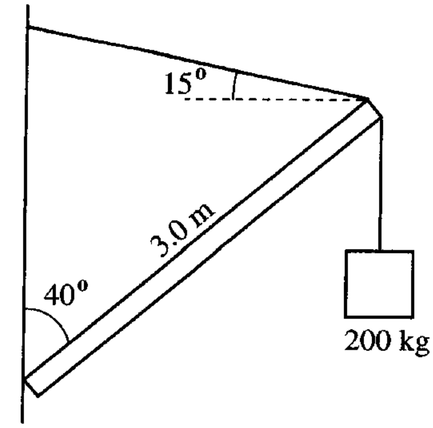
$$= -\frac{3}{4}Mg\cos\theta\sin\theta$$

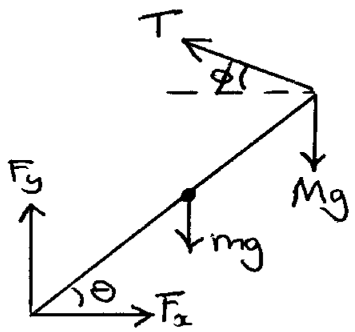
Along y, $F_y - Mg = M a_{cm,y}$

$$F_y = Mg\left(1-\frac{3}{4}\cos^2\theta\right)$$

Along z, $F_z = 0$

9. One end of a thin, uniform beam—of mass 50 kg and length 3.0 m—is attached to a vertical wall. Two massless wires are attached to the other end of the beam. One of the wires is used to support the beam in the orientation shown in the diagram. A 200-kg mass is suspended from the other wire. Find the horizontal and vertical components of the force exerted by the wall on the lower end of the beam.





System is in equilibrium
$\Rightarrow$ Vertical wire has tension equal to weight of suspended mass,

$\sum \vec{F} = \vec{0} \ldots$

along x: $\quad F_x - T\cos\phi = 0 \quad (1)$

along y: $\quad F_y + T\sin\phi - Mg - mg = 0 \quad (2)$

$\sum \vec{\tau} = \vec{0}$ about upper end of beam:

$$F_x L\sin\theta - F_y L\cos\theta + mg\frac{L}{2}\cos\theta = 0 \quad (3)$$

$(1) \times \cos\theta\tan\phi + (2) \times \cos\theta + (3) \div L \Rightarrow$

$$F_x\cos\theta\tan\phi + F_x\sin\theta - (M+m)g\cos\theta + \tfrac{1}{2}mg\cos\theta = 0$$

$$F_x = \frac{(M+\frac{1}{2}m)g}{\tan\theta + \tan\phi}$$

$$= 1540 \text{ N}$$

$(1), (2) \Rightarrow$

$$F_y = (M+m)g - F_x\tan\phi$$

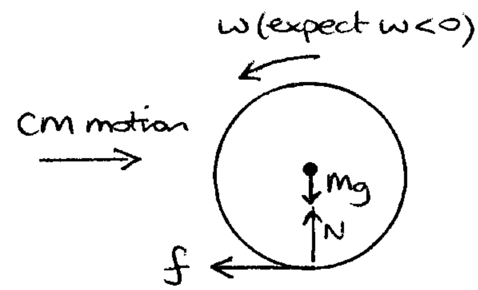
$$= 2090 \text{ N}$$

10. A bowler releases a bowling ball (a uniform sphere of mass M and radius R) so that it lands on the lane moving horizontally at speed v. The ball is not rotating when it hits the lane, and it skids for a certain distance before it begins to roll without slipping. The coefficients of static and kinetic friction between the ball and the surface of the lane are μ_s and μ_k, respectively.

   (a) Find the ball's angular acceleration during the time that it is skidding.

   (b) For what length of time does the ball skid?

   (c) How far from its point of initial contact with the surface does the ball travel before it begins to roll without slipping?



- Newton's laws:

along x: $\quad -f = Ma_{cm} \quad (1)$

along y: $\quad N - Mg = 0 \quad (2)$

about z axis through cm of ball:

$$-fR = I_{cm}\alpha = \frac{2}{5}MR^2\alpha \quad (3)$$

- sliding friction: $\quad f = \mu_k N = \mu_k Mg$ (by (2)) $\quad (4)$

(a) (3), (4) $\Rightarrow$ $\quad \alpha = -\dfrac{5\mu_k g}{2R}$ $\quad$ (constant angular acceleration)

(b) (1), (4) $\Rightarrow$ $\quad a_{cm} = -\mu_k g$ $\quad$ (constant linear acceleration)

Thus $\quad V_{cm} = v - \mu_k g t$

while $\quad \omega = -\dfrac{5\mu_k g}{2R} t$

Rolling without slipping begins when

$$V_{cm} = -R\omega$$

$$\Rightarrow \quad v - \mu_k g t = \frac{5\mu_k g}{2} t$$

$$t = \frac{2v}{7\mu_k g}$$

(c) For constant linear acceleration

$$\Delta x_{cm} = vt + \tfrac{1}{2}a_{cm}t^2 = \frac{2v^2}{7\mu_k g} + \tfrac{1}{2}(-\mu_k g)\left(\frac{2v}{7\mu_k g}\right)^2 = \frac{16v^2}{49\mu_k g}$$