PHZ 3113 Fall 2011 — Exam 1

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all five questions. The maximum possible credit for each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem.

During this exam, you may use one formula sheet. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): _____ Signature: _____

Question	Score
1	
2	
3	
4	
5	
Total	

1. [20 points] Find the interval of convergence of the series $\sum_{n=1}^{\infty}(x-1)^n/\sqrt{n}$. Specify whether or not the series converges at each endpoint of the interval, e.g., distinguish $x > -4$ from $x \geq -4$, and $x < 25$ from $x \leq 25$.

2. [20 points] Consider a gas described by the Dieterici equation of state,

$$p(v-b) = RT \exp\left(-\frac{a}{RTv}\right),$$

where p is the pressure, v is the molar volume, T is the absolute temperature, R is the universal gas constant, and a and b are positive, material-dependent constants.

   (a) Show that this gas has an isothermal compressibility

$$\beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = \frac{1}{p}\,\frac{1-b/v}{1-(1-b/v)a/(RTv)}.$$

   (b) Find the leading two terms in an expansion of β_T in powers of $1/v$ about the limit of infinite volume.

3. [20 points] Evaluate the integral

$$\int_0^{1/2} dx \int_x^{1-x} dy \sqrt{\frac{y-x}{x+y}}$$

by making the transformation $x = (u-v)/2$, $y = (u+v)/2$.

4. [20 points] An above-ground storage tank is to be constructed from steel plate with a horizontal, square bottom of dimensions $b \times b$ and four vertical, rectangular sides of dimensions $b \times h$. The tank will be open to the air at the top. Use the method of Lagrange multipliers to find the ratio h/b that minimizes the amount of steel required to build a tank of any given volume.

5. [20 points] Consider the scalar function $f(x,y,z) = y^2 + 2xz$.

   (a) Estimate the shortest distance that one must travel from $(x,y,z) = (4,0,3)$ in order for f to change by 10^{-3}.

   (b) Find the equation of the tangent plane to the surface of constant f at $(x,y,z) = (3,2,1)$.