**PHZ 3113 Fall 2011 — Exam 2**

**DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO**

**Instructions:** Attempt all five questions. The maximum possible credit for each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem.

During this exam, you may use two formula sheets. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

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Name (printed): ______________________ Signature: ______________________

| Question | Score |
|---|---|
| 1 | ______ |
| 2 | ______ |
| 3 | ______ |
| 4 | ______ |
| 5 | ______ |
| Total | |

1. [15 points] Evaluate

$$\int_0^2 \left[x\delta(x) + x^3\delta(x^2-2) - \Theta(-x) + 3x^2\Theta(x-1)\right] dx,$$

where $\delta(x)$ and $\Theta(x)$ are the Dirac delta function and the Heaviside step function, respectively.

2. [20 points] Use contour-integral methods to evaluate

$$\int_{-\infty}^{\infty} \frac{\cos(\pi x)}{x^2 - 4x + 5}\, dx.$$

3. [25 points]

   (a) Find the roots of the equation $e^{nz} + 1 = 0$ where $n$ is a positive integer.

   (b) Identify the locations $z$ of the isolated singularities of $f(z) = z^2 e^z/(e^{2z}+1)$, and find the residue at each singularity.

4. [20 points] Let $\Phi = \int_A \mathbf{F} \cdot d\mathbf{a}$ be the flux of the vector field $\mathbf{F} = (2x+y)\hat{\mathbf{z}}$ through the surface $A$ corresponding to $z = x(1-x)y(1-y^2)$ with $0 \le x \le 1$, $0 \le y \le 1$, taking the surface element $d\mathbf{a}$ to satisfy $\hat{\mathbf{z}} \cdot d\mathbf{a} \ge 0$. (Here, $\hat{\mathbf{z}}$ is a unit vector along the Cartesian $z$ axis.)

   Use Gauss's theorem to evaluate $\Phi$.

5. [20 points] Use Stokes' theorem to evaluate $\Phi$ defined in Question 4.

   Hint: Express $\mathbf{F}$ as the curl of some vector field.