PHZ 3113 Fall 2011 — Exam 3

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all four questions. The maximum possible credit for each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem.

During this exam, you may use up to three formula sheets. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): ____________________ Signature: ____________________

Question	Score
1	______
2	______
3	______
4	______
Total	

1. [25 points] Recall that the Pauli matrices σ_1, σ_2, and σ_3 satisfy $\sigma_j\sigma_k = I\delta_{jk} + i\sum_{l=1}^{3}\epsilon_{jkl}\sigma_l$, where I is the identity matrix and ϵ_{jkl} is the Levi-Civita symbol. Express the matrix function $\sin(a\sigma_j\sigma_k)$ for real a as a power series, simplifying your result as much as possible. To receive maximum credit, you should express all summations in closed form.

   Hint: You may quote the the power series expansion of $\sin x$, or derive it from $\sin x = (e^{ix} - e^{-ix})/(2i)$.

2. [25 points] Consider the matrix

$$H = \begin{pmatrix} 3 & 2+2i \\ 2-2i & 1 \end{pmatrix} \qquad \text{and a general vector} \qquad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}.$$

   (a) Find the eigenvalues λ_1 and λ_2 of H, defined such that $\lambda_1 < \lambda_2$.

   (b) Find a normalized eigenvector $\mathbf{v}_j$ of H corresponding to each λ_j. Choose each eigenvector so that its first component (the element corresponding to b_1 in $\mathbf{b}$) is real and positive.

   (c) Suppose that we want to express the vector $\mathbf{b}$ with components $b_1 = 2i/3$, $b_2 = (2-i)/3$ in the form $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$. Find c_1 and c_2.

3. [25 points] Find the Fourier sine and cosine series for the function

$$f(x) = \begin{cases} -1 & -\pi < x < -\pi/2, \\ 0 & -\pi/2 < x < \pi/2, \\ 1 & \pi/2 < x < \pi. \end{cases}$$

4. [25 points] A thin, rectangular metallic plate extends over (x, y) coordinates $0 < x < W$ and $0 < y < L$. The temperature T inside the plate satisfies Laplace's equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0.$$

   Along the $y = 0$ and $y = L$ edges, the plate's temperature is held at

$$T(x,0) = T_1 + T_2 \sin\left(\frac{2\pi x}{W}\right) \qquad \text{and} \qquad T(x,L) = T_1 - T_3 \sin\left(\frac{3\pi x}{W}\right).$$

   The other two edges of the plate are held at temperature T_1. Find the full temperature distribution $T(x,y)$ within the plate.

   Hint: Any linear combination of e^{ky} and e^{-ky} can equally well be expressed as a linear combination of $\sinh ky$ and $\sinh k(L-y)$.