PHZ 3113 Fall 2012 — Exam 1

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all five questions. The maximum possible credit for each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem.

During this exam, you may use one formula sheet. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): _____ Signature: _____

Question	Score
1	_____
2	_____
3	_____
4	_____
5	_____
Total	

1. [20 points] Use the method of Lagrange multipliers to find the point (x, y) on the circle $x^2 + y^2 = 4$ that lies farthest from $(3, -1)$.

2. [20 points] Find the interval of convergence of the power series

$$S(x) = \sum_{n=1}^{\infty} \frac{nx^n}{\pi^n (n+r)^2},$$

where r is a positive real number. Specify whether or not the series converges at each endpoint of the interval. Make sure that you note any dependence of the interval of convergence on the value of r.

3. [20 points] Evaluate the integral

$$\int_0^6 dx \int_{x/3}^{x/3+1} dy \;\; x(y - x/3)^2$$

by making the transformation $u = y - x/3$, $v = x/3$.

4. [20 points] The volume thermal expansion coefficient is defined to be

$$\alpha = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p,$$

where p, v, and T are the pressure, molar volume, and absolute temperature, respectively.

   (a) Calculate α for an ideal gas having the equation of state

$$pv = RT.$$

   Express your result as a function of T and/or v but not p.

   (b) Calculate α for a non-ideal gas described by the equation of state

$$pv = RT\left(1 - \frac{v_c}{v}\right),$$

   where v_c is a positive constant. Express your result as a function of T and/or v but not p.

   (c) Based on your answers to (a) and (b), state whether α for the non-ideal gas is smaller or larger than that for the ideal gas in the regime where $v > 2v_c$.

5. [20 points] Suppose that $u = x + y$, $v = x^2 y + xy^2$, and $w = x^3 - y^3$.

   (a) Find $(\partial y/\partial x)_v$ as a function of x and y.

   (b) Find $(\partial u/\partial v)_w$ as a function of x and y.