PHZ 3113 Fall 2012 — Exam 2

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all five questions. The maximum possible credit for each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem.

During this exam, you may use two formula sheets. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): ____________________ Signature: ____________________

Question	Score
1	______
2	______
3	______
4	______
5	______
Total	

Below, $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are unit vectors along the Cartesian x, y, and z axes, respectively. Also, you may find useful the following:

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}, \qquad \cos\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

1. [15 points] Evaluate the flux of the vector field $\mathbf{F} = x^3\hat{\mathbf{x}} + xz^2\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}$ outward through the surface of the cube spanning the region $|x| \leq 1$, $|y-1| \leq 1$, and $|z-2| \leq 1$.

2. [20 points]

   (a) Evaluate
   $$\int_0^5 \left[e^x\, \delta(x^3 - 8) + x\, \Theta(3 - x)\right] dx,$$
   where $\delta(x)$ and $\Theta(x)$ are the Dirac delta function and the Heaviside step function, respectively.

   (b) Evaluate
   $$\int \cos^2\theta\; \delta(r - a)\, dv$$
   where the integration is performed over all of three-dimensional space and (r, θ, ϕ) are standard spherical polar coordinates.

3. [20 points] The two parts of this question are unrelated to one another.

   (a) Given $z_1 = 2e^{i\pi/6}$ and $z_2 = 1 + \sqrt{3}i$, express $(z_1 + z_2)^{1/3}$ in polar notation.

   (b) Evaluate
   $$\oint_C \frac{\sinh z}{3\pi + 2iz}\, dz$$
   taking the integral counterclockwise around a rectangular path C with vertices at $z = \pm 4$ and $z = \pm 4 + 9i$.

4. [20 points] Let
   $$f(z) = \frac{1}{z(z-2)^2}.$$

   (a) Find all nonzero coefficients a_n for $n \leq 1$ in the Laurent series covering the innermost region of analyticity about $z = 2$.

   (b) List all the isolated singularities of $f(z)$, describing each as one of the following: a removable singularity, a pole a certain order (specify the order), or an essential singularity. Find the residue of $f(z)$ at each singularity.

5. [25 points] Use Stokes' theorem to evaluate the flux $\int_A \mathbf{F} \cdot d\mathbf{a}$ of the vector field $\mathbf{F} = (x^2 + y^2)\,\hat{\mathbf{z}}$ through the (open) surface A corresponding to $z = x(4 - x^2)y(1 - y^3)$ with $0 \leq x \leq 2$ and $0 \leq y \leq 1$, taking the surface element $d\mathbf{a}$ to satisfy $\hat{\mathbf{z}} \cdot d\mathbf{a} \geq 0$.

   Hint: Express $\mathbf{F}$ as the curl of some vector.