PHZ 3113 Fall 2012 – Homework 1

Due at the start of class on Friday, August 31. Half credit will be available for homework submitted after the deadline but no later than the start of class on Wednesday, September 5.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. Determine whether or not the following series converge.

   (a) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(n-2)}$,

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{n!(2n)!}{(3n)!}$.

2. Prove that $\displaystyle\sum_{n=1}^{\infty} n^{-p}$ converges for $p > 1$ and diverges for $p \leq 1$.

3. Consider the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

   (a) Show that this series is conditionally convergent.

   (b) Sum the above infinite series by evaluating $\int_0^1 dx/(1+x)$ in two ways: (i) directly; (ii) by expanding the integrand prior to integration.

4. Convert the infinite series defined below to closed-form functions of x by combining the properties of geometric series with integration of series. Specify the interval of x over which your result is valid.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$.

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{x^n}{n(n-1)}$.

5. (a) Find the first four nonvanishing terms in the Taylor expansion of $\ln\bigl(1 + xe^{-x}\bigr)$ about $x = 0$.

   (b) Find the radius of convergence of the expansion in (a).

6. Find the first four nonvanishing terms in the series expansion of $1/x$ about the point $x = 2$ by the following two methods:

   (a) direct construction of the Taylor series about $x = 2$;

   (b) substitution of $y = (x-2)/2$ into the expansion of $1/(1+y)$ about $y = 0$.