PHZ 3113 Fall 2012 – Homework 2

Due at the start of class on Friday, September 7. Half credit will be available for homework submitted after the deadline but no later than the start of class on Monday, September 10.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. In solid state physics, one encounters the function

$$g(x) = \frac{1}{2} - \frac{1-x^2}{4x} \ln\left|\frac{1-x}{1+x}\right|.$$

   (a) Find the leading x dependence of g and of its derivative $g'(x) = dg/dx$ in the limit $0 \le x \ll 1$. What is the sign of g' in this limit?

   (b) Find the leading x dependence of g and of g' in the limit $x \gg 1$. What is the sign of g' in this limit?

   (c) Find the leading x dependence of g and of g' in the limit $|x-1| \ll 1$. What is the sign of g' in this limit?

   (d) Sketch $g(x)$ on a graph that extends from $x = 0$ to $x \gg 1$.

2. If $z = 2xy - x^2$, $x = r\cos\phi$, and $y = r\sin\phi$, find

   (a) the total derivative dz in terms of x, y, dx, and dy;

   (b) the total derivative dz in terms of x, ϕ, dx, and $d\phi$;

   (c) the total derivative dz in terms of r, ϕ, dr, and $d\phi$;

   (d) the partial derivative $(\partial z/\partial x)_y$;

   (e) the partial derivative $(\partial z/\partial x)_\phi$;

   (f) the partial derivative $(\partial r/\partial \phi)_z$.

3. Boas Chapter 4, Section 7, Problem 21 (page 211).

4. We know the molar entropy s of a certain gas as a function of its molar volume v and its absolute temperature T. We also know the molar volume as a function of pressure p and temperature. Show that the molar heat capacities at constant pressure and at constant (molar) volume, $c_p = T(\partial s/\partial T)_p$ and $c_v = T(\partial s/\partial T)_v$, respectively, satisfy

$$c_p = c_v + T\left(\frac{\partial s}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p.$$

   Note: Do not assume any particular equation of state (e.g., the ideal gas law).