PHZ 3113 Fall 2012 – Homework 7

Due at the start of class on Friday, October 19. Half credit will be available for homework submitted after the deadline but no later than the start of class on Monday, October 22.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. A two-dimensional coordinate system (u, v) is related to the standard Cartesian coordinate system by the equations

$$x = \frac{1}{2}(u^2 - v^2), \quad y = uv,$$

where $u > 0$ and $-\infty < v < \infty$. Note that these ranges of u and v allow description of the entire x-y plane except for the origin and the negative-x axis.

   (a) Express u and v as functions of x and y.

   (b) Calculate the metric tensor for the (u, v) coordinate system, and hence show that the system is orthogonal.

   (c) Find the Cartesian equations $y(x)$ or $x(y)$ for the lines $u = U$ and $v = V$, where U and V are constants. Sketch an x-y graph showing (and labeling) the lines $u = \frac{1}{3}$, 1, and 2, as well as the lines $v = 0$, $\pm\frac{1}{3}$, ± 1, and ± 2.

   (d) Find the Cartesian components of the unit vectors $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ at the point $\mathbf{r} = (u, v)$.

   (e) Express an infinitesimal displacement $d\mathbf{r}$ from the point $\mathbf{r} = (u, v)$ in terms of the unit vectors from part (c) and the infinitesimals du and dv.

   (f) Express the gradient operator in the (u, v) coordinate system.

2. Sketch the complex function $z(u) = \ln(ue^{i\pi u/2})$ for real u over the range $\frac{1}{2} \leq u \leq 2$ on a graph of the complex z plane that covers $0 \leq \operatorname{Im} z \leq 6\pi$.

3. Calculate all values of (a) $i^{\,i}$, (b) $i^{(i^{\,i})}$, and (c) $(i^{\,i})^i$. In the last case, show that the results do not satisfy the relation $(a^b)^c = a^{bc}$ that holds for for real a, b, and c.

4. Consider a resistor R, an inductor L, and a capacitor C all placed in parallel with a voltage source $V(t)$.

   (a) Find the combined impedance of the three passive circuit elements (the resistor, the inductor, and the capacitor).

   (b) Given that $V(t) = V_0 \cos \omega t$ where V_0 is real, find the current at time t through each of the four circuit elements (the voltage source and the three passive elements).