PHZ 3113 Fall 2012 – Homework 8

Due at the start of class on Friday, October 26. Half credit will be available for homework submitted after the deadline but no later than the start of class on Monday, October 29.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. Let $f = u + iv$ be an analytic function of $z = x + iy$ with u and v being real functions of x and y. Show that the two-dimensional vector field $\mathbf{F} = u\hat{\mathbf{x}} - v\hat{\mathbf{y}}$ is both solenoidal and irrotational.

   Added note (requiring no work from the student): If $\mathbf{F}$ is solenoidal and irrotational, the same property holds for any field $(u\cos\phi + v\sin\phi)\hat{\mathbf{x}} + (u\sin\phi - v\cos\phi)\hat{\mathbf{y}}$ produced by rotating $\mathbf{F}$ in the x-y plane through an angle ϕ that is independent of x and y. For instance, $\phi = \pi/2$ corresponds to a field $\mathbf{G} = v\hat{\mathbf{x}} + u\hat{\mathbf{y}}$.

2. Use the Cauchy integral formula to evaluate

$$\oint_C \frac{\cosh \pi z}{3z - 4i}\, dz$$

   clockwise around the following contours:

   (a) The circle of radius 1 centered at $z = 0$.

   (b) The circle of radius 1 centered at $z = 2i$.

   (c) The triangle with vertices at $z = 4$, $3i$, and $i - 2$.

3. Find all nonzero coefficients a_n for $|n| \leq 2$ in the Laurent series for

$$f(z) = \frac{z}{(z-1)(z+i)}$$

   within each of the annular regions of analyticity when expanding about (a) $z = 0$, (b) $z = i$. Each coefficient should be expressed in Cartesian form (i.e., $a_n = x_n + iy_n$).

4. For each of the following functions, identify the location, nature (pole of a specified order or essential singularity) and residue of each isolated singularity, assuming that c is a positive real:

   (a) $\dfrac{1}{(z+c)^2}$, (b) $\dfrac{(z+ic)^2}{z^2+c^2}$, (c) $\dfrac{ze^{iz}}{z^2-c^2}$, (d) $\dfrac{c}{(1+\sin z)^2}$.