PHZ 3113 Fall 2012 – Homework 10

Due at the start of class on Wednesday, November 28. Half credit will be available for homework submitted after the deadline but turned no later than the start of class on Friday, November 30.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working. Calculate matrix determinants, inverses, and eigenstates by hand, not with a calculator or computer program.

1. Find the eigenvalues and corresponding orthonormal eigenvectors of the following symmetric or Hermitian matrices.

$$\text{(a)} \begin{pmatrix} 5 & 6 \\ 6 & -3 \end{pmatrix} \quad \text{(b)} \begin{pmatrix} 3 & 1-2i \\ 1+2i & 7 \end{pmatrix} \quad \text{(c)} \begin{pmatrix} 1 & \sqrt{8} & 0 \\ \sqrt{8} & 1 & \sqrt{8} \\ 0 & \sqrt{8} & 1 \end{pmatrix}$$

2. Consider the matrix

$$A = \frac{1}{11} \begin{pmatrix} 2 & 6 & 9 \\ 6 & 7 & -6 \\ 9 & -6 & 2 \end{pmatrix}.$$

   (a) Find the determinant of A and inverse of A, and show that A is orthogonal.

   (b) Find the eigenvalues of A and construct an orthonormal basis of eigenvectors.

3. Find the eigenvalues e_1, e_2 and corresponding normalized eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$ of the non-symmetric matrix

$$\begin{pmatrix} 0 & 3 \\ -2 & 4 \end{pmatrix}.$$

   Check whether the eigenvectors are orthogonal in the sense that $\mathbf{v}_1^\dagger \cdot \mathbf{v}_2 = 0$.

4. Two objects 1 and 2, each of mass m, are attached to opposite ends of a spring of constant $3k$. A second spring, of constant $8k$, connects object 1 to a fixed anchor point. The system is initially in static equilibrium, with object 1 hanging vertically below the anchor point and object 2 hanging vertically below object 1. Then object 2 is pulled vertically downward an extra distance d below its equilibrium point (also causing object 1 to move down a smaller distance) and at time $t = 0$, object 2 is released from rest. Assume that the springs have negligible mass and that all motion is purely vertical.

   (a) Find the equations of motion for the displacements y_1 and y_2 upward from equilibrium. Hint: You can neglect the weights $m_i g$ because in equilibrium, where $y_1 = y_2 = 0$, the weights are canceled by the spring forces.

   (b) Find and solve the characteristic equation for the eigenfrequencies of this system.

   (c) Find the eigenvectors and hence the normal modes of the system.

   (d) Write the configuration of the system at time $t = 0$ as a linear combination of the normal models. In this way, find the values of y_1 and y_2 at a later time $t = T$.