PHZ 3113 Fall 2012 – Homework 11

Due at the start of class on Monday, December 3. Half credit will be available for homework submitted after the deadline but turned no later than the start of class on Wednesday, December 5.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. An ideal full-wave rectifier converts an input current $I_{\text{in}}(t) = I_0 \sin \omega t$ to an output current $I_{\text{full}}(t) = |I_{\text{in}}(t)|$. An ideal half-wave rectifier converts the same input current to an output current $I_{\text{half}}(t) = I_{\text{in}}(t)\, \Theta(I_{\text{in}}(t))$, where $\Theta(x)$ is the Heaviside function.

   (a) Find the Fourier sine and cosine series for the output of the full-wave rectifier.

   (b) Find the Fourier sine and cosine series for the output of the half-wave rectifier. Hint: To reduce your work, examine the quantity $I_{\text{half}}(t) - \frac{1}{2} I_{\text{full}}(t)$.

2. The Fourier series for both of the following functions $f(x)$ was derived in class. Use each of these Fourier series to evaluate the corresponding sum S.

   (a) $f(x) = x^2$ for $|x| < \pi$; $S = \sum_{k=1}^{\infty} (-1)^{k-1}\, k^{-2}$.

   (b) $f(x) = 0$ for $-\pi < x < 0$, $f(x) = 1$ for $0 < x < \pi$; $S = \sum_{k=1}^{\infty} (-1)^{k-1}\, (2k+1)^{-1}$.

3. Calculate the Fourier transform of $f(x) = x^2 \Theta(w - |x|)$, where $w > 0$ is a real constant and $\Theta(x)$ is the Heaviside function. Hint: Use integration by parts.

4. Consider a solution of the differential equation $d^2y/dx^2 + x\, dy/dx + y = 0$ in the series form $y = \sum_{n=0}^{\infty} a_n x^{n+s}$ with $a_0 \neq 0$.

   (a) Calculate and solve the indicial equation to find two allowed values of s, the lowest power of x present in the solution.

   (b) For each allowed value of s, find a recurrence relation for the coefficients a_n and hence find a_n/a_0.

   (c) For each allowed value of s, find the interval of convergence of the corresponding series solution.

5. Consider a solution of the differential equation $x(x-1)d^2y/dx^2 + 3x\, dy/dx + y = 0$ in the series form $y = \sum_{n=0}^{\infty} a_n x^{n+s}$ with $a_0 \neq 0$.

   (a) Calculate and solve the indicial equation to find two allowed values of s, the lowest power of x present in the solution.

   (b) For each allowed value of s, find a recurrence relation for the coefficients a_n.

   (c) Show that one of the two allowed values of s leads to a series solution with well-defined a_n/a_0 that converges within some interval of x.

   (d) Show that the other allowed value of s does not lead to a valid series solution because a_1/a_0 is infinite.