PHY 3513 Fall 2000 – Homework 1

Due at the start of class on Friday, September 8.

Answer all questions. To obtain full credit, you must explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. (a) Suppose that 10.0 g of ice initially at −20.0 °C is dropped into 30.0 g of water initially at 40.0 °C. Assuming that the combined ice-water system is closed (i.e., no material or heat crosses its boundaries and no work is done by/on the system), what is the final equilibrium state? Express your final answer in roughly the same format as the following sample answers (all of which are physically nonsensical): "40.0 g of ice at −30.0 °C," "5.0 g of ice at −20.0 °C plus 35.0 g of water at 5.0 °C," and "20.0 g of water and 20.0 g of steam, both at 100 °C."

   (b) Redo part (a) for a situation in which 10.0 g of ice initially at −20.0 °C is brought into contact with 30.0 g of steam initially at 110.0 °C.

   Data: $c_{\text{ice}} = 2220\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$, $c_{\text{water}} = 4190\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$, $c_{\text{steam}} = 1520\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$,
   $L_{\text{F}} = 333 \times 10^3\,\text{J}\,\text{kg}^{-1}$, $L_{\text{V}} = 2256 \times 10^3\,\text{J}\,\text{kg}^{-1}$, $T_{\text{melt}} = 0\,°\text{C}$, $T_{\text{boil}} = 100\,°\text{C}$.

2. As mentioned in class, the molar heat capacity c_{mol} of most substances approaches a universal constant value at high temperatures. At low temperatures, however, c_{mol} varies greatly from material to material, and is also temperature dependent. For many metals, the molar heat capacity is well-described by the low-temperature form

   $$c_{\text{mol}} = \gamma T + \alpha T^3, \tag{1}$$

   where T is the *absolute* temperature; both γ and α depend on the metal in question.

   Integrate the equation

   $$đQ = n\, c_{\text{mol}}\, dT,$$

   where n is the number of moles, to find the total heat Q when 0.4 mol of a metal described by Eq. (1) is cooled from 25 K to 12 K. Take $\gamma = 0.9\,\text{mJ}\,\text{mol}^{-1}\text{K}^{-2}$ and $\alpha = 0.2\,\text{mJ}\,\text{mol}^{-1}\text{K}^{-4}$.

3. A certain sample of gas is sealed inside a cylinder with a movable piston. The pressure P, the volume V, and the temperature T of the gas are related by the equation of state

   $$P(V - a) + \frac{c}{V - b} = d\,T. \tag{2}$$

   Here a, b, c, and d are positive constants.

   Calculate the total work done by the gas on expanding from volume V_1 to $V_2 > V_1$ when the expansion is performed in each of three different ways specified by the various constraints specified in (a)–(c) below.

   (a) Isobaric: $P = P_0$, a constant.

   (c) Isothermal: $T = T_0$, a constant.

   (c) Isochoric: $V = V_1 = V_2$.

   Hint: You may find it convenient to use partial fractions to re-express $\dfrac{1}{(V-a)(V-b)}$.

4. One mole of an ideal gas undergoes the following three-step cycle, starting from pressure P_0 and volume V_0:
(i) An isochoric cooling during which the absolute temperature of the gas is halved.
(ii) An isothermal expansion to a volume V_2.
(iii) An adiabatic return to the starting point of the cycle.

(a) Draw a P-V diagram (horizontal axis = V) showing the path followed by the system as it undergoes this closed thermodynamic cycle. Be sure to label the three steps (i, ii, and iii) and draw arrows to show the direction that the system moves on the diagram.

(b) Calculate the value of V_2 in terms of other variables introduced above and the adiabatic exponent γ.

(c) Calculate the work performed by the system during each step of the cycle and hence the total work performed during the complete cycle.

(d) Evaluate the lowest temperature reached during the cycle and the total work performed, given that $P_0 = 2.0 \times 10^5\,\text{Pa}$ and $V_0 = 1.0 \times 10^{-2}\,\text{m}^3$. Assume $\gamma = 5/3$.

Comments

- Problem 1 is rather unusual for a physics problem. Usually, you are encouraged to derive a purely purely algebraic equation for the quantity or quantities that you are asked to find; you are advised to plug in numerical values for algebraic symbols on the right hand side of your equation(s) only at the very end of the calculation. This method has significant advantages: the algebraic result can be more readily checked for dimensional consistency, and you can also test your equation in various special cases (such as when certain input quantities vanish); once verified, the same equation can be applied repeatedly by substituting different sets of numerical data. For these reasons, the "all-algebra" method should be used wherever possible.

  Problem 1 provides an example where this approach doesn't work. In order to get to the correct answer, you have to pose and answer a series of qualitative questions such as "Does the ice warm up to 0 °C before the water cools to 0 °C?" or "Does all the water freeze before the ice reaches 0 °C?" The sequence of questions that you face will depend on the answers to previous questions; rather than waiting until the end of the problem to plug in numbers, you have to use numbers at every stage of the decision process in order to know which question to ask next.

- Problems 2 and 3 are designed mainly to test your understanding of fundamental thermodynamic quantities (including their all-important signs) and your ability to apply integration in a physics context. The integrals in Problem 2 should be easy for you, while those in Problem 3 may be a little harder. You will be expected to be able to do integrals such as these throughout the course, both as part of homework and during assembly exams.

- Problem 4 introduces the idea of a thermodynamic cycle. We will spend a lot of time thinking about cycles in Callen Ch. 4.