PHY 3513 Fall 2000 – Homework 2

Due at the start of class on Friday, September 15.

Answer all questions. To obtain full credit, you must explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. The kinetic theory of gases as described in class ignores the effect of gravity on the molecules as they traverse their container. This problem is designed to give you some idea whether this is a serious omission.

   Consider a cubic container, 10 cm along each side (i.e., a volume of 1 liter), filled with helium gas held at 1.0 atm pressure and a temperature of 20 °C.

   (a) Calculate the mean kinetic energy per molecule, ignoring any effect of gravity.

   (b) Calculate the change in gravitational potential energy of one molecule when it rises from the bottom of the container to the top.

   (c) Assuming that the mean total (kinetic plus gravitational potential) energy is constant throughout the gas, estimate the implied temperature difference between the bottom and top of the container.

2. In 5.0 sec, 150 bullets strike and embed themselves in a wall. The bullets, each of mass 30 g, strike the wall perpendicularly at a speed of 1100 m/s.

   (a) What is the average change in momentum per second for the entire set of bullets?

   (b) Determine the average force exerted on the wall during the 5-second period.

   (c) Assuming that the bullets are spread out evenly over an area of 2.5×10^{-4} m^2, obtain the average pressure they exert on the wall during the 5-second period.

3. In class we computed the entropy change during the free expansion of n moles of ideal gas from initial volume V_i to final volume V_f. Using the fact that S is a state function, we computed ΔS for the free expansion (an irreversible process) by looking at an isothermal expansion (a reversible process).

   Repeat the calculation of ΔS for the free expansion of an ideal gas, this time using a two-step reversible process between the initial and final states: (i) an isobaric expansion, followed by (ii) an adiabatic cooling. On a P–V diagram, sketch and label the two-step process, and also show the isothermal path between the same endpoints.

   Useful fact: An ideal gas obeying $U = cnRT$ has an adiabatic exponent $\gamma = 1 + 1/c$.

4. Consider a box containing a gas of N point-like molecules which do not interact with one another. The box is considered to be divided into two equal parts. (The division is imaginary — no physical barrier separates the parts.)

   Let $P_N(N_L)$ be the probability of finding exactly N_L of the N molecules in the left half of the box. We saw in class that $P_N(N_L)$ is peaked at $N_L = \frac{1}{2}N$. The goal of this problem is to investigate how the width of the peak depends on N.

   Calculate (a) $P_N(\frac{1}{2}N - 1)/P_N(\frac{1}{2}N)$, (b) $P_N(\frac{1}{4}N)/P_N(\frac{1}{2}N)$, and (c) $P_N(1)/P_N(\frac{1}{2}N)$, for each of three cases: (i) $N = 20$, (ii) $N = 40$, (iii) $N = 60$.

   Note: Do not use Stirling's approximation. Most scientific calculators are able to compute N directly for $N \leq 69$.