

## PHY 3513 Fall 2000 – Exam 2: The Recount

\_\_\_\_\_ : Your Exam 2 scores were  $\square + \square + \square = \square$  (out of 60).

The assignment this week offers you the opportunity to improve your score on the exam. You may turn in a new solution to any or all questions that appeared on the exam (see annotated copy below). For each question that you submit, the effective score used in calculating your course grade will be the mean of the old and new scores (rounded up to the nearest integer).

**You must turn in your solutions in person to Kevin Ingersent by 5:00 p.m. on Friday, December 1.** (Do not leave solutions in Kevin's departmental mailbox or push them under his office door.)

## PHY 3513 Fall 2000 – Mid-Term Exam 2

*Hints and comments have been added by italicizing parts of the original questions and by inserting new text in italics.*

Answer all three questions. The maximum score on each question is as shown. To obtain full credit, you must explain your reasoning and show all working. You may quote without proof any equation from Callen, provided that you give the equation number. (*Pay attention to these requirements — they will be enforced!*) Please write neatly and include your name on the front page of your answers.

*General hint: Make sure that quantities that should be extensive are indeed extensive, and those that should be intensive do turn out to be intensive; if not, go back and find your error.*

1. [Max. 15 points] A thermodynamic system obeys the equations

$$T = \frac{As}{v^{1/2}}, \quad P = \frac{Bs^2}{v^{3/2}}. \quad (1)$$

- (a) By taking appropriate partial derivatives of Eqs. (1), find the relation between the constants  $A$  and  $B$  that ensures thermodynamic consistency. Use your result to replace  $B$  in the second equation. ( *$B$  should not enter your answers to the remaining parts of the problem.*)
  - (b) Integrate Eqs. (1) to find the fundamental relation  $u(s, v)$ . *Follow the rules given in class, and check by taking partial derivatives of your answer that you recover Eqs. (1).*
  - (c) Calculate the molar chemical potential  $\mu(s, v)$ . *Avoid the Gibbs-Duhem relation!*
  - (d) Calculate the ratio of molar heat capacities  $c_P/c_v$  for this system. (Your result should be a constant, *in fact, it should be a rational number, such as  $\frac{1}{2}$  or 10.*)
2. [Max. 15 points] This question compares two different ways of bringing  $N_1$  moles of gaseous helium into thermal equilibrium with  $N_2$  moles of gaseous hydrogen. The gases are contained in separate vessels, the volumes of which remain constant during equilibration. The initial temperatures of the helium and the hydrogen are  $T_1$  and  $T_2$ , respectively. Over the temperature range of interest, helium and hydrogen can be treated as simple ideal gases with  $U/NRT = c_1$  and  $c_2$ , respectively. *The purpose of the preceding sentence is to tell you to use Callen Eq. (3.26) with  $c = c_1$  for helium and  $c = c_2$  for helium;  $c_1$  and  $c_2$  do not represent heat capacities or molar heat capacities.*

Process I: The two vessels are placed next to each other in such a way that heat can flow through the wall separating the gases. Assume that the gases together form a closed system and that the thermal conductivity of the separating wall is sufficiently low that the process can be treated as quasistatic.

Process II: Instead, the two gases are brought into thermal equilibrium in such a way as to deliver the maximum possible work to a reversible work source. (A third thermodynamic subsystem would be required to carry out such a process. *However, you should not include this third system explicitly in any of your equations.*)

- (a) Find  $T_I$ , the equilibrium temperature of the gases at the end of Process I.
  - (b) Calculate the change in entropy of the gases during Process I. The variable  $T_I$  may appear in your answer.
  - (c) Find  $T_{II}$ , the equilibrium temperature of the gases at the end of Process II.
  - (d) Calculate the work delivered to the reversible work source during Process II. The variable  $T_{II}$  may appear in your answer.
  - (e) Explain on general grounds whether  $T_{II}$  should be less than, greater than, or equal to  $T_I$ .
3. [*Max. 30 points*] Consider an evacuated cylinder, within which the electromagnetic radiation obeys the fundamental relation

$$S = \frac{4}{3}(bU^3V)^{1/4}.$$

- (a) Calculate the pressure of the radiation as a function of  $T$  and  $V$ .
- (b) Calculate the entropy of the radiation as a function of  $T$  and  $V$ .
- (c) Calculate the entropy of the radiation as a function of  $P$  and  $V$ .
- (d) Calculate the relation between  $P$  and  $V$  for this system during a quasistatic adiabatic process. *Don't assume that you can apply the result for an ideal gas.*

The evacuated cylinder is used as the auxiliary system in a Carnot cycle, operating between a minimum temperature  $T_c$  and a maximum temperature  $T_h$ , and between a minimum volume  $V_s$  and a maximum volume  $V_b$ .

Note: The quantities requested in parts (f)–(i) below should be expressed in terms of  $b$ ,  $T_h$ ,  $T_c$ ,  $V_s$ , and  $V_b$  (and no other variables, *such as*  $V_A$ ,  $V_B$ ,  $V_C$ , and  $V_D$ ).

- (e) Draw a  $P$ – $V$  diagram for the auxiliary system, with  $V$  plotted along the horizontal axis. Label the four vertices  $A$ – $D$ , using the convention followed in class and in Callen (i.e.,  $A$  is the start of the heat intake step). Indicate the relation between  $P$  and  $V$  for each step, e.g., “ $PV^5 = \text{const}$ ” or “ $V = \text{const}$ ” (*not* “ $T = \text{const}$ ”).
- (f) Calculate  $V_A$ ,  $V_B$ ,  $V_C$ , and  $V_D$ .
- (g) Calculate  $Q_{AB}$ , the heat entering the auxiliary system during the first step of the Carnot cycle.
- (h) Calculate  $W_{AB}$ , the work done *on* the auxiliary system during the first step of the Carnot cycle.
- (i) Calculate  $W$ , the total work done *on* the auxiliary system during one complete Carnot cycle.

Hint: Although you can calculate  $W$  as the sum of the work for each step of the Carnot cycle, you are advised to seek a less time-consuming method.