PHY 3513 Fall 2000 – Information Concerning Mid-Term Exam 2

- The second mid-term will take place between 7:00 and 9:00 pm on Tuesday November 21. The exam will be held in room NPB 1220.
- The exam will focus on material relating to Callen Chapters 3 and 4. However, due to the cumulative nature of the course, the problems may also touch on earlier topics. You will not be tested on Callen Sections 3.8, 4.8, or 4.9. In lieu of Sections 4.1–4.3 you should familiarize yourself with the the discussion of quasistatic and reversible processes given in lectures. Reversible heat engines will be covered, but not refrigerators, heat pumps, or irreversible heat engines.
- You should bring a scientific calculator and pens/pencils to the exam. You will be permitted to use the course text (Callen) and your lecture notes in the exam. However, you must not consult any other written materials, such as homework solutions (either your own or others').
- The sample exam below is designed to give an idea of the level of the questions on the Mid-Term. The sample exam will not be graded. Solutions will be distributed in class.

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This exam lasts 2 hours. Answer all four questions, which carry equal weight. To obtain full credit, please explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

You are permitted to use the course text (Callen) and your lecture notes, but you may **not** *consult any other written materials (e.g., homework solutions).*

You must not seek or obtain help on this exam from anyone other than the proctor, nor must you assist anyone else.

1. The tension τ of a particular rubber band is found to obey the equations

$$\left(\frac{\partial \tau}{\partial T}\right)_L = \frac{aL}{L_0}\left[1 - \left(\frac{L_0}{L}\right)^3\right], \qquad \left(\frac{\partial \tau}{\partial L}\right)_T = \frac{aT}{L_0}\left[1 + 2\left(\frac{L_0}{L}\right)^3\right],$$

where T is the temperature, L is the length, L_0 is the unstretched length (at which the tension vanishes), and a is a positive constant.

(a) Verify by calculating the mixed second partial derivatives of the tension that there exists a well-defined state function $\tau(T, L)$.

(b) Integrate the two partial derivatives above to obtain $\tau(T, L)$. There should be no undetermined constant in your answer.

Hints: (1) You should not use any of the results in Section 3.7 of Callen, which describes a slightly simpler model for a rubber band. (2) The procedure for integrating partial derivatives of the tension to get $\tau(T, L)$ is essentially the same as that for integrating equations of state to get the fundamental equation.

2. The electromagnetic radiation inside a closed cavity of volume V obeys the fundamental equation

$$S = \frac{4}{3}(bU^3V)^{1/4}.$$

(a) Calculate the constant-volume heat capacity $C_V(V, T)$.

(b) Calculate the adiabatic compressibility $\kappa_S(S, P)$.

(c) Calculate $P(V, T)$. Hence show that the constant-pressure heat capacity C_P and the thermal expansion coefficient α are both equal to zero.

3. A fixed quantity of water, initially at temperature T_w, has a heat capacity C_w which can be taken to be independent of temperature.

   (a) Suppose that the water is brought into contact with a thermal reservoir at temperature T_r, where $T_r > T_w$. Assuming quasistatic heat transfer, how much has the entropy of the entire system changed by the time the water has come to equilibrium with the reservoir?

   (b) Suppose instead that the water is brought into equilibrium first with a thermal reservoir at T_m, then with a second reservoir at T_r, where $T_r > T_m > T_w$. Once again assuming quasistatic heat transfer, what is the overall entropy change for the entire system?

   (c) For fixed C, T_w, and T_r, what value of T_m minimizes the overall entropy change for the two-step process described in (b)?

4. The air-standard diesel cycle consists of the following four steps: (i) adiabatic compression from volume V_A to V_B; (ii) isobaric expansion from volume V_B to V_C; (iii) adiabatic expansion from volume V_C to V_D; (iv) an isochoric return to the starting point. Callen Fig. 4.11 illustrates this cycle on P–V and T–S diagrams.

   Consider an implementation of this cycle, using for the auxiliary system a fixed amount of a simple ideal gas described by the fundamental equation

$$s = s_0 + R \ln \left[\left(\frac{u}{u_0} \right)^{1/(\gamma - 1)} \frac{v}{v_0} \right].$$

   Calculate the engine efficiency for this implementation. Assume that each step in the cycle is carried out quasistatically. Express your final answer solely in terms of the parameter γ and the volumes V_A, V_B, and V_C.