PHY 3513 Fall 2000 – Information Concerning Final Exam

- The final term will take place between 12:30 and 2:30 p.m. on Friday, December 15. The exam will be held in room NPB 1220.
- The exam will focus on material relating to Callen Chapters 1-7. The following sections are "fair game":
  - Chapter 1, Sections 3–6, and 8–10: Extensive variables, systems and walls, equilibrium states, the entropy postulates.
  - Chapter 2, Sections 1–8: Intensive variables, equations of state, thermal, mechanical and diffusional equilibrium between subsystems.
  - Chapter 3, Sections 1–7 and 9: Euler equation, Gibbs-Duhem relation, integrating equations of state, ideal and van der Waals gases, cavity radiation, rubber band, second derivatives of the fundamental relation.
  - Chapter 4, Sections 2, 4–7 and 10: quasistatic and reversible processes (focus on what was covered in lectures), heat flow between systems, maximum work, heat engines, refrigerators, and heat pumps, the Carnot cycle, other reversible cycles.
  - Chapter 5, Sections 1 and 3: energy minimum formulation of thermodynamics, thermodynamic potentials and their partial derivatives.
  - Chapter 6, Sections 1 and 2: Helmholtz potential minimum principle, and its applications.
  - Chapter 7, Sections 1 and 2: Maxwell relations (but not mnemonic diagrams).
- You should bring a scientific calculator and pens/pencils to the exam. You will be permitted to use the course text (Callen) and your lecture notes in the exam. However, you must not consult any other written materials, such as homework solutions (either your own or others').
- The sample exam below is designed to give an idea of the level of the questions on the Final. The sample exam will not be graded. Solutions will be distributed in class.

PHY 3513 Fall 2000 – Sample Final Exam

This exam lasts 2 hours. Answer all questions. The maximum points for each question or part question are shown in brackets. To obtain full credit, please explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

You are permitted to use the course text (Callen) and your lecture notes, but you may **not** *consult any other written materials (e.g., homework solutions).*

You must not seek or obtain help on this exam from anyone other than the proctor, nor must you assist anyone else.

1. [35 points] Each part of this problem is completely independent of the other parts.

   (a) One mole of an ideal gas A and one mole of a van der Waals gas B are contained in a rigid cylinder and are separated by a movable internal piston. The system is held at a constant temperature T. When the piston is released and allowed to come to equilibrium, it is found that the two gases occupy equal volumes, $V_A = V_B = V$. Find an expression for V in terms of T, the parameters a and b of the van der Waals gas, and universal constants. [7 points]

(b) A system obeys the fundamental relation

$$s = s_0 + A(vu^2)^{1/4}.$$

Express u as a function of T and P. [7 points]

(c) If we regard the molar internal energy of a simple one-component system as a function of T and v, then the First Law of Thermodynamics becomes

$$du = T\left(\frac{\partial s}{\partial T}\right)_v dT + \left[T\left(\frac{\partial s}{\partial v}\right)_T - P\right] dv.$$

Using this equation as a starting point, calculate $(\partial u/\partial v)_T$ for a gas described by the equation

$$Pv = RT[1 + B(T)/v].$$

Hint: Use a Mawxwell relation to replace one of the partial derivatives of s with a partial derivative of P, which you can then evaluate. [7 points]

(d) A simple one-component system is found to obey the equations

$$T = \sqrt{\frac{P}{a^2bv}}\exp(-bv^2), \quad P = 4buv,$$

where a and b are constants. What is the fundamental equation $s = s(u, v)$ for this system? [7 points]

(e) Two subsystems, having heat capacities C_1 = constant and $C_2 = \gamma_2 T$, start at temperatures T_{1i} and T_{2i}, respectively. Find the final common temperature T_f when the subsystems are placed in thermal contact to form a closed composite system. Express your result in terms of T_{1i}, T_{2i} and the ratio C_1/γ_2. [7 points]

2. [13 points] A simple ideal gas satisfying $Pv = RT$ and $u = cRT$ (where c may be taken to be a constant) is used as the auxiliary system of a heat engine. During one cycle of the engine, the gas traces a rectangle on the P–V diagram, starting at the bottom left-hand corner, according to the following sequence: $(V_1, P_1) \to (V_1, P_2) \to (V_2, P_2) \to (V_2, P_1) \to (V_1, P_1)$.

   (a) For each step of the cycle, calculate the change in internal energy of the gas ΔU, the heat transfer into the gas Q, and the work done on the gas W. During which step or steps does heat flow into the gas? [10 points]

   (b) Calculate the overall values of ΔU, Q and W for one complete cycle. [3 points]

3. [12 points.] A system is described by the molar Helmholtz free energy

$$f(T, v) = RT\ln\left(1 - e^{-A/T}\right), \quad A = \theta_0 - \theta_1 v/v_0. \tag{1}$$

   (a) Calculate the molar entropy $s(T, v)$ and the pressure $P(T, v)$. [6 points]

   (b) Calculate the molar energy $u(T, v)$. [3 points]

   (d) Calculate the fundamental relation $s = s(u, v)$. [3 points]