PHY 3513 Fall 1998 – Homework 10

Due at the start of class on Friday, November 20.

Answer all questions. To obtain full credit, please explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. Callen Problem 4.4-1.

2. Callen Problem 4.4-6.

   Hint: You may neglect the change in the temperature of any vat, but **not** the change in its entropy.

   You should find that for $N \to \infty$ it becomes possible to transfer heat to a body and raise its temperature without increasing its entropy. This result should be contrasted with the "continuous free expansion" considered in class, where the entropy increases even in the limit of infinitesimal expansion steps.

3. Callen Problem 4.5-6.

4. Callen Problem 4.5-1,2.

   In the first problem you should use the maximum work theorem; in the second, you should calculate the work and heat transfers directly.

5. Callen Problem 4.7-4.

6. Consider a Carnot engine in which the hot and cold reversible heat sources (RHS's) have finite, temperature-independent heat capacities, C_h and C_c, respectively; C_c is several (2–3) times bigger than C_h. The auxiliary system (AS) operates between $S = S_i$ and $S = S_i + dS$.

   Let the initial temperatures of the RHS's be $T_{h,i}$ and $T_{c,i}$ and assume that the auxiliary system (AS) starts at $S = S_i$, $T = T_{h,i}$.

   (a) Draw a T-S diagram for the AS, showing the first 5 steps of the Carnot cycle. Be careful to show the relative slopes of the lines in a qualitatively correct fashion.

   (b) Assuming that dS is an infinitesimal quantity, find the common temperature T_f at which the hot and cold RHS eventually equilibrate.

   (c) Find the total heat drawn from the hot RHS before it reaches equilibrium with the cold RHS.

   (d) Find the total reversible work output to the reversible work source.

   (e) Find the overall engine efficiency.

7. The Otto cycle for a simple ideal gas (for which $U = cNRT$) was discussed in class. Assume that the auxiliary system (AS) in a particular heat engine operates this cycle between volumes V_A and $V_B < V_A$, and between minimum and maximum temperatures T_A and T_C, respectively. (See Callen Fig. 4.9 for the labeling of points on the cycle.)

   (a) Draw a P-V diagram for the AS.

   (b) Calculate W, the work done by the AS per cycle, as a function of symbols introduced above.

   (c) Re-express the result of (b) in terms of the engine efficiency ϵ_e derived in class, and plot W as a function of ϵ_e.

   (d) Given that there is a theoretical maximum value for ϵ_e, what is the maximum value of W, and for what value of V_B/V_A does it occur?