PHY 3513 Fall 1998 – Take-Home Final Exam

Answer all four questions, which carry equal weight. To obtain full credit, please explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

You must not seek or obtain help on this exam from anyone other than the instructor, nor must you assist anyone else.

The exam should be turned in to Kevin Ingersent in person or pushed under his office door (*not left in his mailbox)* by 6 pm on Wednesday, December 16.

1. The velocity at which sound propagates through a fluid is

$$v_s = (\rho \kappa_s)^{-1/2}, \tag{1}$$

where $\rho = m/v$ is the mass density (m being the molar mass) and $\kappa_s = -v^{-1}(\partial v/\partial P)_s$ is the adiabatic compressibility.

   (a) Calculate $\kappa_s(T, v)$ for an ideal van der Waals fluid, described by the fundamental equation

$$s = s_0 + R \ln \left[(v - b)(u + a/v)^c \right],$$

   where a and b are positive constants.

   Hint: As intermediate steps, obtain expressions for $P(s, v)$ and hence $\kappa_s(s, v)$.

   (b) Determine $v_{s,\mathrm{vdW}}(T, v)$, the velocity of sound in the van der Waals fluid.

   (c) By setting $a = 0$ and $b = 0$, obtain $v_{s,\mathrm{ideal}}$, the velocity of sound in the corresponding simple ideal gas. Evaluate the high-temperature limit of $v_{s,\mathrm{vdW}}/v_{s,\mathrm{ideal}} - 1$. Give a brief qualitative explanation of the sign of your answer.

2. A system obeys the equations of state

$$u = av^{1/3}T, \quad 3Pv = (s + bv)T.$$

   (a) Find the fundamental relation $s = s(u, v)$. (Don't worry if your answer violates the Nernst postulate.)

   (b) Calculate the molar chemical potential $\mu(u, v)$.

3. The absorption refrigerator is an alternative to the conventional refrigerator. The energy driving the transfer of heat from temperature T_c to T_h is supplied not as input work, but rather as heat flow from a source (typically a gas flame) at temperature $T_f > T_h > T_c$. No work whatsoever is involved.

   (a) Draw a diagram similar to Callen Fig. 4.6(b) showing the heat flows in the absorption refrigerator. Be careful to stick to the convention that heat added to a system is positive.

   (b) Calculate the ideal efficiency of the absorption refrigerator, defined as the ratio of the heat extracted at $T = T_c$ to the heat supplied by the source at $T = T_f$ under reversible operating conditions. Your answer should involve only the three temperatures T_c, T_h, and T_f.

(c) It is instructive to compare the absorption refrigerator with a composite thermodynamic device consisting of (i) a heat engine operating between T_f and T_h; and (ii) a conventional refrigerator operating between T_c and T_h. All the work output by the heat engine is supplied as work input to the refrigerator.

Draw a diagram similar to that in (a) for the composite device.

(d) Show that the ideal efficiency of the composite device, defined in the same way as in (b), is identical to that of the absorption refrigerator.

4. A system is described by the molar Helmholtz free energy

$$f(T, v) = RT \ln \left(1 - e^{-A/T}\right), \quad A = \theta_0 - \theta_1 v/v_0. \tag{2}$$

(a) Calculate the molar entropy $s(T, v)$ and the pressure $P(T, v)$.

(b) Sketch the temperature variation of s and P at fixed v.

Hint: To get the curves qualitatively correct, it may help to find the forms of s and P in the limits $T \ll A$ and $T \gg A$.

(c) Calculate the molar energy $u(T, v)$.

(d) Calculate the fundamental relation $s = s(u, v)$.