PHY 4523 Spring 2001 – Homework 3

Due at the start of class on Friday, February 16.

To gain full credit you should explain your reasoning and show all working. Please write neatly and remember to include your name on the front page of your answers.

Consider a system, composed of N classical spins, which is in thermal equilibrium with a heat reservoir at temperature T, and which is subjected to a uniform magnetic field of strength H. The spins are distinguishable and independent of one another. Each spin has precisely three possible orientations, directed at angles $\theta = 0, \pm 2\pi/3$ to the direction of the magnetic field. The magnetic energy of each spin is $\epsilon(\theta) = -\mu H \cos\theta$, where μ is the magnetic moment, a (known) constant.

(a) Write down the partition function for this system. Neglect all non-magnetic degrees of freedom, i.e., just take into account the magnetic energy for each spin.

(b) Calculate the Helmholtz free energy F of the system.

(c) The total magnetic energy of the system can be written $E_{\mathrm{mag}} = -MH$, where M is the magnetization. Calculate M by using the total differential $dF = -S\,dT - M\,dH$.

(d) Derive the limiting forms for M in the limits $k_B T \ll \mu H$ and $k_B T \gg \mu H$. In each case, include the leading nonvanishing dependence on both T and H.

(e) Sketch M as a function of H at fixed T. You should pay special attention to getting the low-field and high-field limits qualitatively correct.