PHY 4523 Spring 2001

Information Concerning the Final Exam

The final exam will be held on Tuesday, May 1 from 3:00 to 5:00 p.m. in 1011 NPB. You will need a calculator and a writing implement during the exam. You may also bring your lecture notes, Reif and/or Callen, but no other written materials.

The questions will focus on topics from the following list:

- Quantum statistics of ideal gases—Maxwell-Boltzmann, Planck, Bose-Einstein, and Fermi-Dirac distributions.
- Ideal gases—microstates, density of states, partition function in the classical limit, rotational and vibrational modes of diatomic gases.
- The photon gas—cavity radiation, black-body radiation.
- Ideal Fermi fluids—conduction electrons in metals.
- Lattice vibrations—normal modes of 1D monatomic chains, Einstein model, Debye model.
- Ferromagnetism—mean-field treatment of the Ising model.

Sample Final Exam

You have two hours in which to attempt **four** *of the following five problems, each of which is worth the same maximum number of points. To gain full credit, explain your reasoning and show all working. You are allowed to consult Reif, Callen, and/or your lecture notes.*

1. A dilute gas consists of N diatomic molecules confined to a volume V. Each molecule consists of two identical nuclei, each of mass m. When rotating about an axis through the center of mass and perpendicular to the line joining the two nuclei, the molecule has a moment of inertia I. Consider the temperature range $T_{class} \ll T \ll \hbar^2/(2Ik_B)$, where T_{class} is the temperature above which the center-of-mass motion can be treated classically.

   (a) Calculate the partition function in this limit. Take into account both the translational and rotational degrees of freedom.

   (b) Use your answer to (a) to evaluate the constant-volume heat capacity.

2. The vibrational energy levels of a diatomic molecule are given by $\epsilon_n = (n + \frac{1}{2})\hbar\omega$, $n = 0, 1, \ldots$

   (a) Calculate $P_{odd}(T)$, the probability at temperature T that the molecule is in a mode of odd quantum number ($n = 1, 3, 5, \ldots$). Note: P_{odd} is the **combined** probability that the molecule is in any of the odd-n modes (not the probability that it is in one mode n, where n happens to be odd).

(b) Calculate the low-temperature limit of $P_{odd}(T)$. Include the leading T-dependent correction about the $T = 0$ result.

(c) Calculate the high-temperature form of $P_{odd}(T)$, including the leading T-dependent correction about the limit $T = \infty$.

3. Consider the electromagnetic radiation inside a closed cavity of volume V held at a uniform temperature T.

   (a) Calculate the constant-volume heat capacity of the radiation. Express your answer as a function of V, the temperature T, universal constants, and numerical factors.

   (b) Express the radiation pressure as a function of T, universal constants and numerical factors.

   (c) Determine the constant-pressure heat capacity, $C_P = T(\partial S/\partial T)_P$. Hint: Think before you calculate!

   (d) Calculate the average number of photons inside the cavity as a function of T, universal constants and numerical factors. You may leave an integral over a dimensionless variable in your answer. Hint: Figure out how to get the number of photons with frequency between ω and $\omega + d\omega$, then integrate over ω.

4. Consider an *ultra-relativistic* gas of N noninteracting electrons in a cubic box of sides L and volume $V = L^3$. Just as in the nonrelativistic case, the single-particle states have wavevectors of the form

$$\mathbf{k} = \frac{2\pi}{L}(n_x, n_y, n_z),$$

   where n_x, n_y, and n_z are integers. However, in the ultra-relativistic limit, the energy of each state can be approximated by

$$\epsilon(\mathbf{k}) = \hbar c|\mathbf{k}|,$$

   where c is the velocity of light.

   (a) Calculate the Fermi energy ϵ_F.

   (b) Calculate the constant-volume heat capacity in the limit of high temperatures $T \gg \epsilon_F/k_B$, where the Fermi-Dirac distribution can be approximated by the Maxwell-Boltzmann distribution. Express your result as a function of N, k_B, and numerical constants. Hint: Evaluate Reif Eqs. (9.17.1) and (9.17.2) with $F(\epsilon)$ and $\rho(\epsilon)$ suitably modified to describe the problem at hand. You may find Reif Eq. (A.3.3) useful in evaluating the integrals.

5. The velocity of sound waves in liquid ^{4}He at temperatures below 0.6 K is 238 m/s. The sound waves are longitudinal—there are no transverse modes. The density of liquid ^{4}He is 145 kg/m^3 and the mass of one ^{4}He atom is 6.7×10^{-27} kg.

   (a) Calculate the Debye temperature for liquid ^{4}He.

   (b) Calculate the constant-volume specific heat capacity of liquid ^{4}He at 0.3 K (in units of J kg^{-1}K^{-1}).