PHY 4604 Fall 2008 — Exam 2

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all three questions. The maximum possible credit for each part of each question is shown in square brackets. Please try to write neatly!

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem and $\hbar$ (the reduced Planck constant).

During this exam, you may use two formula sheets. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): ______________________ Signature: ______________________

Question	Score
1	____
2	____
3	____
Total	

1. [14 points total] A particle of mass m moves in one dimension under the potential $V(x) = -Fx$, where $F > 0$ is real.

   (a) [5 points] Write down the Schrödinger wave equation for the momentum-space stationary state of energy E.

   (b) [5 points] Solve the wave equation from part (a) to obtain the momentum-space wave function in terms of one unknown constant. You may assume without proof that the solution of the differential equation $dy/dx = f(x)\, y$ can be written $y(x) = y(0) \exp[\int_0^x f(x')dx']$.

   (c) [4 points] Show that the momentum probability density in this state is independent of momentum.

2. [40 points total] A particle of mass m moves in one dimension under the real-valued potential $V(x) = V_1\Theta(x) - aV_0\delta(x)$, where $a > 0$ and $V_0 > 0$, but V_1 may be of either sign. $\Theta(x) = 0$ for $x < 0$ and $\Theta(x) = 1$ for $x > 0$.

   (a) [6 points] Write down the form of the bound-state wave function $\psi(x)$ in the regions $x < 0$ and $x > 0$. You may leave in your answer any unknown amplitude that may have a nonzero value in a physically acceptable state. All other symbols that don't appear in the statement of the problem should be defined.

   (b) [12 points] By applying the appropriate boundary conditions, obtain an equation relating the bound-state energy E to other quantities defined above. Express your answer in the form $f(E) =$ constant.

   (c) [10 points] Determine the range of V_1 over which a bound state exists.

   (d) [12 points] Find the bound-state energy E in closed form. You should be able to eliminate all square roots from the equation you found in (b) by squaring both sides, then carrying some terms from one side to the other side before squaring again. For $V_1 = 0$, the energy should reduce to $E = E_0 = -ma^2V_0^2/(2\hbar^2)$, the standard result for a pure delta-function potential $V(x) = -aV_0\delta(x)$.

3. [46 points total] A quantum-mechanical system is described by a two-dimensional vector space spanned by orthonormal basis vectors $|1\rangle$ and $|2\rangle$. Here, $|1\rangle$ and $|2\rangle$ are eigenvectors of an observable operator $\hat{G}$ with eigenvalues $g_1 = \gamma$ and $g_2 = 2\gamma$, respectively. The Hamiltonian for this system is $\hat{H} = 2\epsilon\,(|1\rangle\langle 1| + |2\rangle\langle 2|) + 3i\epsilon\,(|1\rangle\langle 2| - |2\rangle\langle 1|)$. Both γ and ϵ are positive.

   (a) [6 points] Express $\hat{G}$ and $\hat{H}$ as matrices in the basis $\{|1\rangle, |2\rangle\}$.

   (b) [12 points] Find the eigenvalues E_1 and E_2 of $\hat{H}$ and express the corresponding eigenkets $|E_1\rangle$ and $|E_2\rangle$ as linear combinations of $|1\rangle$ and $|2\rangle$. (It is advisable to check your answers by calculating $\hat{H}|E_j\rangle$ using the outer-product form of $\hat{H}$.)

   (c) [20 points] At time zero, the system is described by a state vector $e^{i\pi/3}|1\rangle$. What are the possible outcomes of a measurement of the observable G performed at time $t > 0$? Give the probability of each outcome.

   (d) [8 points] For the situation described in (c), at what times $t > 0$ could the measurement be performed to maximize the expectation value of G?