PHY 4604 Fall 2009 — Exam 1

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all three questions. The maximum possible credit for each part of each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem and $\hbar$ (the reduced Planck constant).

During this exam, you may use one formula sheet. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): ____________________ Signature: ____________________

Question	Score
1	______
2	______
3	______
Total	

You may find useful the following formulae:

$$\sin 2x = 2\cos x \sin x \qquad \cos^2 x + \sin^2 x = 1,$$

$$\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x \qquad \int \cos^3 x \, dx = \sin x - \frac{1}{3}\sin^3 x$$

1. A particle of mass m moves under the influence of the one-dimensional potential that takes the value V_0 for $0 < x < a/2$, the value 0 for $a/2 < x < a$, and is infinite everywhere else.

   Plot the wave functions $\psi_a(x)$ and $\psi_b(x)$ defined in (a) and (b) below on the same graph, using the same horizontal and vertical scales. Make sure that you label $x = 0$, $x = a/2$, and $x = a$ on the horizontal axis. The graph should make clear how the two wave functions differ with regard to qualitative features such as (i) the relative amplitudes of the wave functions in each spatial regions, and (ii) the relative wavelengths in those regions where the probability density is oscillatory.

   (a) [8 points] First consider the case $V_0 = 0$ in which one has an infinite square well. Sketch the $n = 4$ stationary state wave function (under the convention that $n = 1$ represents the ground state). Take the wave function to be real, and label it "$\psi_a(x)$" on the graph.

   (b) [16 points] Now consider the case $V_0 = 5\hbar^2/(ma^2)$. Again, sketch the $n = 4$ stationary state wave function (where $n = 1$ represents the ground state). **Do not** attempt a quantitative solution for the detailed form of this wave function. Take the wave function to be real, and label it "$\psi_b(x)$" on the graph. If $\psi_b(x)$ crosses $\psi_a(x)$, make sure that it is clear which curve is which.

2. A particle moves in the one-dimensional potential $V(x) = 0$ for $0 < x < a$, $V(x) = \infty$ for $x < 0$ or $x > a$. At a certain moment, the system is described by a wave function $\Psi(x) = \sqrt{2/(5a)}\,[2\sin(\pi x/a) - \sin(2\pi x/a)]$ for $0 \le x \le a$.

   (a) [10 points] What is the expectation value of the particle's energy at this moment?

   (b) [10 points] What is the uncertainty in the particle's energy at this moment?

   (c) [20 points] What is the probability that a measurement of the particle's position performed at this moment will yield a result $x < a/2$?

3. A particle of mass m moves in the one-dimensional potential $V(x) = \frac{1}{2}m\omega^2 x^2$. At time 0, the system's wave function is $\Psi(x, 0) = \frac{1}{\sqrt{2}}[\psi_1(x) + \psi_2(x)]$, where $\psi_n(x)$ ($n = 0$, 1, 2, ...) is the n^{th} stationary state as conventionally defined.

   (a) [6 points] What is the wave function at time $t \ge 0$?

   (b) [15 points] Calculate the mean and standard deviation of the distribution of the results of a position measurement performed at time $t \ge 0$. For what value(s) of t does the mean value of x vanish?

   (c) [15 points] Calculate the mean and standard deviation of the distribution of the results of a momentum measurement performed at time $t \ge 0$. For what value(s) of t does the mean value of p vanish?