PHY 4604 Fall 2010 — Exam 1

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all three questions. The maximum possible credit for each part of each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem and $\hbar$ (the reduced Planck constant).

During this exam, you may use one formula sheet. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): ____________________ Signature: ____________________

Question	Score
1	______
2	______
3	______
Total	

1. [30 points total] A one-dimensional harmonic oscillator of mass m and angular frequency ω is in an initial state $\Psi(x, t=0) = \left[i\,\psi_1(x) - \sqrt{2}\,\psi_3(x)\right]/\sqrt{3}$, where $\psi_n(x)$ is the n^{th} stationary state as conventionally defined.

   (a) [6 points] Express the wave function at time $t = T > 0$ in terms of ψ_n's.

   (b) [8 points] List the possible results of an accurate measurement of the oscillator's total energy performed at time $t = T$, as well as the probability of each result.

   (c) [16 points] Calculate the mean and standard deviation of the oscillator's momentum at time $t = T$.

2. [30 points total] A particle of mass m moves in one dimension under the influence of the potential

$$V(x) = \begin{cases} V_0 & \text{for } x < 0, \\ 0 & \text{for } x > 0, \end{cases}$$

   with $V_0 = \hbar^2/(ma^2)$. The particle is in a stationary state $\psi(x)$ of energy $E = V_0/2$.

   (a) [8 points] Write $\psi(x < 0)$ in terms of known quantities and one or more unknown amplitudes. Find the wavelength or the exponential decay length of $\psi(x < 0)$.

   (b) [8 points] Write $\psi(x > 0)$ in terms of known quantities and one or more unknown amplitudes. Find the wavelength or the exponential decay length of $\psi(x > 0)$.

   (c) [14 points] Sketch a graph of the probability density $\rho(x)$ for $-2a \le x \le 6a$. **Do not** attempt a full quantitative description of $\rho(x)$. However, you should accurately convey the characteristic length scales over which the probability density varies. Label the points $x = -2a$, $x = 0$, and $x = 6a$ on the horizontal axis.

3. [40 points total] A particle of mass m particle is confined in one dimension by a potential $V(x)$ that is zero for $|x| < L$ and is infinite everywhere else. At time $t = 0$, the particle is in a state described by the normalized wave function

$$\Psi(x, t=0) = \begin{cases} \sqrt{\dfrac{105}{16L^7}}\left(L^2 x - x^3\right) & \text{for } |x| < L, \\ 0 & \text{otherwise.} \end{cases}$$

   Try to minimize the number of integrals you perform in answering parts (a)–(e) below.

   (a) [7 points] What is the expectation value of the particle's position at $t = 0$?

   (b) [7 points] What is the expectation value of the particle's momentum at $t = 0$?

   (c) [10 points] What is the uncertainty in the momentum at $t = 0$?

   (d) [8 points] What is the smallest value that could be obtained in an accurate measurement of the particle's total energy at time $t = 0$?

   (e) [8 points] The system evolves without outside disturbance from $t = 0$ to $t = T > 0$. The questions in (a)–(d) above can be posed at $t = T$ instead of at $t = 0$. **Do not** attempt to answer these questions at $t = T$. Instead, explain for each part (a)–(d) whether the answer at $t = T$ must be the same as at $t = 0$ or whether it can be different.