PHY 4604 Fall 2010 — Exam 2

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt both questions. The maximum possible credit for each part of each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every part of each question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem and $\hbar$ (the reduced Planck constant).

During this exam, you may use two formula sheets. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): ____________________ Signature: ____________________

Question	Score
1	____
2	____
Total	

1. [50 points total] A particle of mass m moves in the one-dimensional potential

$$V(x) = -\frac{v\hbar^2}{2md}\left[\delta\left(x-d\right)+\delta\left(x+d\right)\right],$$

where $d > 0$ is a distance and $v > 0$ is a dimensionless real number. This question focuses on the ground state of the problem: a stationary state $\psi_0(x)\exp(-iE_0t/\hbar)$ having energy $E_0 = -\hbar^2K^2/2m < 0$. Since $V(-x) = V(x)$, the ground state necessarily has even parity under $x \to -x$, i.e., $\psi_0(x) = \psi_0(-x)$.

   (a) [12 points] Write down the general form of $\psi_0(x)$ within each region of constant potential. Express your answer in terms of variables defined above and unknown amplitudes. Minimize the number of such amplitudes in your answer by (i) enforcing the even parity of the wave function, and (ii) including only terms whose amplitude can possibly be nonzero in a normalizable state.

   (b) [10 points] Apply appropriate boundary conditions to obtain two equations linking K defined above and the unknown amplitudes entering your answer to (a).

   (c) [6 points] Sketch the qualitative form of $|\psi_0|^2$ vs x over the range $-3d < x < 3d$, under the assumption that Kd is a number of order 1.

   (d) [8 points] Show that the equations you obtained in (b) can be combined to yield the transcendental equation

$$\frac{2Kd}{1+e^{-2Kd}} = v. \tag{1}$$

   (e) [6 points] Sketch the left-hand side of Eq. (1) as a function of K (or, if you prefer, $2Kd$), and hence show that there is a solution K of the equation for any $v > 0$.

   (f) [8 points] Use Eq. (1) to find an expression for the bound state energy, valid in the limit $v \ll 1$. Your answer should include the leading dependence of E_0 on v.

2. [50 points total] A quantum-mechanical system is described by a two-dimensional vector space spanned by orthonormal basis vectors $\{|1\rangle, |2\rangle\}$. The Hamiltonian operator for this system is $\hat{H} = 3\epsilon|2\rangle\langle 2| + 2i\epsilon(|2\rangle\langle 1| - |1\rangle\langle 2|)$. Another observable operator is $\hat{A} = \alpha|1\rangle\langle 1| + 3\alpha|2\rangle\langle 2|$. Here, ϵ and α are positive, real quantities.

   (a) [8 points] Express $\hat{H}$ and $\hat{A}$ as matrices in the basis $\{|1\rangle, |2\rangle\}$.

   (b) [12 points] Find the eigenvalues E_1 and E_2 $(> E_1)$ of $\hat{H}$. Express the corresponding eigenkets $|E_1\rangle$ and $|E_2\rangle$ as linear combinations of $|1\rangle$ and $|2\rangle$. Choose the overall phase of each eigenket so that the amplitude of $|1\rangle$ is a positive, real number.

   (c) [15 points] At time zero, the system is described by a state vector $(2|1\rangle - i|2\rangle)/\sqrt{5}$. Find the state vector $|\psi(t)\rangle$ at time $t > 0$ as a linear combination of $|1\rangle$ and $|2\rangle$.

   (d) [15 points] What are the possible outcomes of a measurement of A performed at time $t > 0$ on the state defined in part (c)? What is the probability of each outcome?