PHY 4604 Spring 2012 — Exam 1

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all three questions. The maximum possible credit for each part of each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem and $\hbar$ (the reduced Planck constant).

During this exam, you may use one formula sheet. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): ____________________ Signature: ____________________

Question	Score
1	______
2	______
3	______
Total	

Note: $\displaystyle\int \cos^2 x\, dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin 2x, \qquad 2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta).$

1. At time $t = t_0$, a particle of mass m confined to an infinite square well spanning $|x| < a/2$ has normalized wave function $\Psi(x,t_0) = \sqrt{2/(25a)}[3\cos(\pi x/a) + 4\cos(3\pi x/a)]$ inside the well.

   (a) [20 points] Find the expectation values of the particle's position, momentum, and energy at $t = t_0$.

   (b) [20 points] Find the probability that a measurement of the particle's position at $t = t_0$ yields a result between $-a/4$ and $a/4$. What is the ratio of this probability to the corresponding one for a classical particle moving in the same potential?

2. A harmonic oscillator of mass m and angular frequency ω is in an initial state $\Psi(x, t = 0) = \cos(\theta)\,\psi_1(x) + e^{i\phi}\sin(\theta)\,\psi_2(x)$, where θ and ϕ are real, and $\psi_n(x)$ ($n = 0, 1, \ldots$) is the n^{th} time-independent wave function as conventionally defined.

   (a) [7 points] Express the wave function at time $t \geq 0$ in terms of ψ_n's.

   (b) [18 points] Find the position and momentum expectation values at time $t \geq 0$.

   (c) [5 points] Find a θ that maximizes the amplitude of the oscillations of the position expectation value.

   (d) [5 points] Find a ϕ such that the position expectation value vanishes at $t = 0$.

3. A particle of mass m moves in the one-dimensional potential $V(x) = V_0 x/L$ for $0 < x < L$; $V(x) = \infty$ for $x < 0$ or $x > L$. **Plot the wave functions $\psi_a(x)$ and $\psi_b(x)$ defined in (a) and (b) below on the same graph**, using the same horizontal and vertical scales. Make sure that you label $x = 0$, $x = L/4$, $x = L/2$, $x = 3L/4$, and $x = L$ on the horizontal axis. The graph should make clear how the two wave functions differ with regard to qualitative features such as (i) the relative amplitudes of the wave functions in different parts of the potential well, and (ii) the relative locations of the nodes.

   (a) [5 points] Consider first the case $V_0 = 0$ for which the problem reduces to a standard particle in a box. Sketch and label the normalized spatial wave function $\psi_a(x)$ with energy E_a describing the third excited bound state, i.e., the state having three nodes in the region $0 < x < L$.

   (b) [20 points] Now consider a case $V_0 \approx E_a/2$. Sketch and label the normalized wave function $\psi_b(x)$ describing the third excited bound state. You should deduce the form of $\psi_b(x)$ by referring to the shooting method and by drawing analogies with classical physics to answer the following questions:

      i. How many nodes will $\psi_b(x)$ have in the region $0 < x < L$?

      ii. Will the nodes of $\psi_b(x)$ be located to the left of, to the right of, or precisely at the positions of the corresponding nodes of $\psi_a(x)$? Explain your reasoning.

      iii. Do you expect that the maximum value of $|\psi_b(x)|$ over the interval $0 < x < L/2$ to be greater than, less than, or the same as the maximum value of $|\psi_b(x)|$ over the interval $L/2 < x < L$? Explain your reasoning.