PHY 4604 Spring 2012 — Exam 2

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt both questions. The maximum possible credit for each part of each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every part of each question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem and $\hbar$ (the reduced Planck constant).

During this exam, you may use two formula sheets. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): ______________________ Signature: ______________________

Question	Score
1	______
2	______
Total	

1. [50 points total] A quantum-mechanical system is described by a two-dimensional vector space spanned by orthonormal basis vectors $\{|1\rangle, |2\rangle\}$. In this basis, the Hamiltonian $\hat{H}$ and another observable operator $\hat{C}$ have matrix representations

$$\hat{H} = \epsilon \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}, \qquad \hat{C} = \gamma \begin{pmatrix} 4 & 3 \\ 3 & -4 \end{pmatrix}.$$

Here, ϵ and γ are positive, real quantities.

(a) [8 points] Express $\hat{H}$ and $\hat{C}$ in outer-product form in the basis $\{|1\rangle, |2\rangle\}$.

(b) [12 points] Find the eigenvalues E_1 and E_2 of $\hat{H}$ (choosing $E_1 < E_2$) and express the corresponding eigenkets $|E_1\rangle$ and $|E_2\rangle$ as linear combinations of $|1\rangle$ and $|2\rangle$. Choose the overall phase of each eigenket so that the amplitude of $|1\rangle$ is a positive, real number.

(c) [10 points] At time zero, the system is described by a state vector $|2\rangle$. Express the state vector at time $t > 0$ as a linear combination of $|1\rangle$ and $|2\rangle$.

(d) [20 points] What are the possible outcomes of a measurement of C performed at time $t = 0$ on the system defined in part (c)? Give the probability of each outcome.

2. [50 points total] A particle of mass m moves in the one-dimensional potential

$$V(x) = \begin{cases} V_0\, a\, \delta(x) & \text{for } |x| < a/2, \\ \infty & \text{for } |x| > a/2. \end{cases}$$

Since $V(x) = V(-x)$, any non-degenerate solution $\psi(x)$ of the time-independent Schrödinger equation (TISE) must be either even or odd under reflection about $x = 0$.

(a) [8 points] Write down the boundary conditions that any solution $\psi(x)$ of the TISE must obey at the locations $x = 0$ and $x = \pm a/2$ of jumps in $V(x)$.

(b) [8 points] Consider the special case $V_0 = 0$ for which the potential reduces to an infinite square well. Write down the normalized solutions $\psi_n(x)$ of the TISE and their energies $E_n(V_0 = 0)$, where $n = 1, 2, 3, \ldots$ and $E_{n+1} > E_n$.

(c) [8 points] Identify those states from (b) that remain solutions of the TISE for any $V_0 > 0$. Give the energy $E_n(V_0)$ of any such state.

(d) [16 points] Show that any solution $\psi_n(x)$ of the TISE for $V_0 > 0$ that does not belong to the set identified in (c) can be written (up to a multiplicative normalization constant A_n) as a trigonometric function of the argument $k_n(a/2 - |x|)$, where k_n is a root of the equation

$$\tan\left(\frac{k_n a}{2}\right) = -\frac{\hbar^2 k_n}{m V_0 a}.$$

(e) [10 points] Use $\operatorname{atan}(1/\alpha) \simeq (2j+1)\pi/2 - \alpha + O(\alpha^3)$ for integer j to approximate k_n in (d) to first order in $m V_0 a^2/\hbar^2$ when the latter is a small quantity. Hence, evaluate the energy shift $E_n(V_0) - E_n(0)$ resulting from the delta function term in the potential. Simplify your final answer as much as possible.