PHY 4604 Spring 2013 — Exam 1

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Instructions: Attempt all three questions. The maximum possible credit for each part of each question is shown in square brackets. Please try to write your solution neatly and legibly.

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem and $\hbar$ (the reduced Planck constant).

During this exam, you may use one formula sheet. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

Name (printed): _____ Signature: _____

Question	Score
1	_____
2	_____
3	_____
Total	

You may find useful the following formulae:

$$\sin 2x = 2\cos x \sin x \qquad \cos^2 x + \sin^2 x = 1,$$

$$\int \sin^2 x\, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x \qquad \int \cos^3 x\, dx = \sin x - \frac{1}{3}\sin^3 x$$

1. [30 points total] A one-dimensional harmonic oscillator of mass m and angular frequency ω is in an initial state $\Psi(x, t = 0) = \big[\psi_0(x) - i\psi_2(x)\big]/\sqrt{2}$, where $\psi_n(x)$ is the nth stationary state as conventionally defined.

   (a) [6 points] Express the wave function at time $t = T > 0$ in terms of ψ_n's and other quantities defined in the problem.

   (b) [16 points] Calculate the mean and standard deviation of the oscillator's position x at time $t = T$.

   (c) [8 points] List the mean and standard deviation of the oscillator's total energy E at time $t = T$.

2. [40 points total] A particle of mass m moves under the influence of the one-dimensional potential $V(x)$ that takes the value $V_0 > 0$ for $|x| < a/2$, the value $-V_0$ for $a/2 < |x| < a$, and the value 0 for $|x| > a$. The particle has bound states $n = 0, 1, 2, \ldots$, described by real spatial wave functions $\psi_n(x)$ and energies $E_n < 0$, as well as scattering states having positive energies. This question concerns the first excited bound state, $n = 1$.

   (a) [8 points] Write $\psi_1(x)$ in the region $x > a$ in terms of quantities defined above and one or more unknown amplitudes.

   (b) [8 points] Write $\psi_1(x)$ in the region $a/2 < x < a$ in terms of quantities defined above and one or more unknown amplitudes.

   (c) [8 points] Write $\psi_1(x)$ in the region $-a/2 < x < a/2$ in terms of quantities defined above and one or more unknown amplitudes.

   (d) [16 points] Sketch a graph of $\psi_1(x)$ spanning the range $-2a < x < 2a$, assuming that $E_1 \approx -V_0/2$ and $V_0 \approx \pi^2\hbar^2/(4ma^2)$. **Do not** attempt a full quantitative description of $\psi_1(x)$. However, you should convey the characteristic length scales over which the wave function varies. Label the points $x = 0$, $x = \pm a/2$, and $x = \pm a$ on the horizontal axis.

3. [30 points total] A particle moves in the one-dimensional potential $V(x) = 0$ for $0 < x < a$, $V(x) = \infty$ for $x < 0$ or $x > a$. At a certain moment, the system is described by a wave function $\Psi(x) = \sqrt{2/(5a)}\,[\sin(\pi x/a) - 2\sin(2\pi x/a)]$ for $0 \le x \le a$.

   (a) [6 points] What is the expectation value of the particle's energy at this moment?

   (b) [6 points] What is the uncertainty in the particle's energy at this moment?

   (c) [18 points] What is the probability that a measurement of the particle's position performed at this moment will yield a result $x < a/4$?