**PHY 4604 Spring 2013 — Exam 2**

**DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO**

**Instructions:** Attempt both questions. The maximum possible credit for each part of each question is shown in square brackets. Please try to write neatly!

You will receive credit only for knowledge and understanding that you demonstrate in your written solutions. It is in your best interest to write down something relevant for every question, even if you can't provide a complete answer. To maximize your score, you should briefly explain your reasoning and show all working. Give all final algebraic answers in terms of variables defined in the problem and $\hbar$ (the reduced Planck constant).

During this exam, you may use two formula sheets. You are not permitted (a) to consult any other books, notes, or papers, (b) to use any electronic device, or (c) to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Print your name where indicated below, and sign to confirm that you have read and understood these instructions. Please do not write anything else below the line.

---

Name (printed): ____________________ Signature: ____________________

| Question | Score |
|---|---|
| 1 | ____ |
| 2 | ____ |
| Total | |

1. [55 points total] A quantum-mechanical system is described by a two-dimensional vector space spanned by orthonormal basis vectors $|1\rangle$ and $|2\rangle$. In this basis, the Hamiltonian $\hat{H}$ and another observable operator $\hat{G}$ have the matrix representations

$$\hat{H} = \epsilon \begin{pmatrix} 3 & -2i \\ 2i & 3 \end{pmatrix}, \qquad \hat{G} = \gamma \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}.$$

Both $\epsilon$ and $\gamma$ are positive reals.

   (a) [8 points] Express $\hat{G}$ and $\hat{H}$ in outer-product form in the basis $\{|1\rangle, |2\rangle\}$.

   (b) [15 points] Find the eigenvalues $E_1$ and $E_2$ ($> E_1$) of $\hat{H}$ and express the corresponding eigenkets $|E_1\rangle$ and $|E_2\rangle$ as linear combinations of $|1\rangle$ and $|2\rangle$. Choose the phase of each $|E_j\rangle$ so that the amplitude of $|1\rangle$ is a positive real.

   (c) [15 points] At time zero, the system is described by a state vector $e^{i\pi/3}|2\rangle$. Express the state vector at time $t > 0$ as a linear combination of $|1\rangle$ and $|2\rangle$.

   (d) [17 points] For the situation described in (c), what are the possible outcomes of a measurement of the observable $G$ performed at time $t > 0$? At what times (plural) $t > 0$ will the expectation value of $G$ be smallest?

2. [45 points total] A particle of mass $m$ moves in one dimension under the real-valued potential $V(x) = S\Theta(x) - \alpha\delta(x)$, where $a$, $S$, and $\alpha$ are all positive, real numbers; $\Theta(x) = 0$ for $x < 0$ and $\Theta(x) = 1$ for $x > 0$.

   (a) [4 points] What is the range within which the energy $E$ of a bound state for this potential must lie on general grounds?

   (b) [10 points] Write down the form of a bound-state wave function $\psi(x)$ in the regions $x < 0$ and $x > 0$. You may leave in your answer any unknown amplitude that may have a nonzero value in a physically acceptable state. All other symbols that don't appear in the statement of the problem should be defined.

   (c) [12 points] By applying the appropriate boundary conditions, obtain an equation relating the bound-state energy $E$ to other quantities defined above. Express your answer in the form $f(E) = \text{constant}$.

   (d) [7 points] Show that no bound state can exist for $S \geq S_{\text{max}} = 2m\alpha^2/\hbar^2$.

   (e) [12 points] Assuming that $S < S_{\text{max}}$, find the bound-state energy $E$ in closed form. You should be able to eliminate all square roots from the equation you found in (c) by squaring both sides, then carrying some terms from one side to the other side before squaring again. For $S = 0$, the energy should reduce to $E = -S_{\text{max}}/4 = -m\alpha^2/(2\hbar^2)$, the standard result for a pure delta-function potential $V(x) = -\alpha\delta(x)$.