PHY 6645 Fall 2001 – Homework 1

Due at the start of class on Thursday, September 6.

Answer all questions. To gain full credit you should explain your reasoning and show all working. Please write neatly and remember to include your name on the front page of your answers.

1. Ballentine Problem 1.2: Consider the vector space that consists of all possible linear combinations of the following functions: 1, $\sin x$, $\cos x$, $(\sin x)^2$, $(\cos x)^2$, $\sin(2x)$, and $\cos(2x)$. What is the dimension of this space? Exhibit a possible set of basis vectors, and demonstrate that it is complete.

   Note: A basis is *complete* if every vector in the vector space can be expressed as a linear combination of the basis vectors.

2. The set of 2×2 square matrices A having complex entries A_{ij} form a four-dimensional vector space under the normal rules of matrix addition and scalar multiplication.

   Construct a complete basis for this space. What is the null vector?

   Which of the following sets constitutes a subspace of this vector space? Explain your reasoning. For any set that is a subspace, state the dimension and provide a complete basis.

   (a) The set of traceless matrices satisfying $\sum_i A_{ii} = 0$.

   (b) The set of Hermitian matrices.

   (c) The set of unitary matrices.

3. Shankar Ex. 1.6.2 (p. 27)

4. Shankar Ex. 1.6.4 and 1.6.5 (p. 29)

5. Shankar Ex. 1.7.1 (p. 30)