PHY 6645 Fall 2001 – Homework 10

Due by noon on Tuesday, December 11. No credit will be available for solutions submitted after this deadline. Please turn in your solutions in person to Donna Balkcom, NPB 2152. Do NOT place them in Kevin Ingersent's mailbox or office.

Answer all questions. To gain full credit you should explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. Shankar Exercise 14.3.4.

2. Shankar Exercise 14.3.7.

3. A beam of electrons is partially polarized by a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{x}}$. The Hamiltonian for each electron can be written

$$H = \frac{1}{2m}\mathbf{P}\cdot\mathbf{P} - \mu\mathbf{S}\cdot\mathbf{B},$$

where $\mathbf{P}$ and $\mathbf{S}$ are the momentum and spin operators, and μ is a (positive, real) constant. Assume that the electrons are in thermal equilibrium at temperature T. Then quantum statistical mechanics tells us that the density matrix is

$$\rho = A\exp(-H/k_BT)$$

where k_B is Boltzmann's constant and A is a normalization factor.

   (a) Show that the density operator for this system factorizes into orbital and spin parts: $\rho = \rho_o \otimes \rho_s$.
   (b) Calculate the spin density matrix for this system in the basis of eigenstates of S_z.
   (c) Find the ensemble expectation value $\langle\mathbf{S}\rangle$ and the magnitude of the system's polarization.
   (d) Find the eigenstates of the spin density operator, and also find $\langle\mathbf{S}\rangle$ in each eigenstate.

4. Ballentine Problem 7.9: Consider a system of two spin-$\frac{1}{2}$ particles. Calculate the eigenvalues and eigenvectors of the operator $\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$. Use the product vectors $|m_1\rangle\otimes|m_2\rangle$ as basis vectors.

5. Shankar Exercise 15.1.2.

6. Suppose that $\mathbf{A}$ and $\mathbf{B}$ are vector operators. Prove that $\mathbf{A}\cdot\mathbf{B}$ commutes with J_x, J_y, and J_z.

7. Based on Sakurai Problem 3.20: We are to add angular momenta $j_1 = 1$ and $j_2 = 1$ to form $j = 2$, 1, and 0 states. Express each of the nine $\{j, m\}$ eigenkets as a sum of product basis states $|j_1, m_1; j_2, m_2\rangle$. You should derive the relevant Clebsch-Gordan coefficients, **not** merely quote them.

   You are, of course, free to check the correctness of your answers against precalculated CG coefficients. A handy one-page table of CG coefficients is available in Postscript format at `http://pdg.lbl.gov/2001/clebrpp.ps` and in PDF format at `http://pdg.lbl.gov/2001/clebrpp.pdf`. An online CG calculator is available at `http://www.ph.surrey.ac.uk/~phs3ps/cleb.html`, which also has links to several other calculators.