

PHY 6645 Fall 2001 – Homework 2

**Due at the start of class on Thursday, September 20. No credit will be available for solutions submitted after 4 p.m. on Friday, September 21.**

*Answer all questions. To gain full credit you should explain your reasoning and show all working. Please write neatly and remember to include your name on the front page of your answers.*

Note: If you want to practice more problems on some of the material covered in this homework set, any or all of Shankar Exercises 1.8.1–9 and 1.9.1–3 are worth attempting. Don't turn in solutions to these additional problems for grading, but feel free to consult Kevin Ingersent if you have questions.

1. Find the eigenvalues and eigenvectors for each of the matrices below:

$$(a) \begin{pmatrix} 5 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{pmatrix}, \quad (b) \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 3i \\ 0 & -3i & 0 \end{pmatrix}, \quad (c) \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}.$$

2. Simplify  $e^{a\Lambda}\Omega e^{-a\Lambda}$  (where  $a$  is complex) for the following special cases:

$$(a) [\Lambda, \Omega] = \hbar, \quad (b) [\Lambda, \Omega] = \hbar\Lambda, \quad (c) [\Lambda, \Omega] = \hbar\Omega, \quad (d) [\Lambda, \Omega] = 0.$$

3. Expand  $[\Lambda^2, \Omega^2]$  as a sum of terms of the form  $C\Omega^m\Lambda^n$ , where  $m$  and  $n$  are nonnegative integers, and each  $C$  is either  $[\Lambda, \Omega]$  or a higher-order commutator of the form  $[\dots[\cdot, [\Lambda, \Omega]]\dots]$ . Note that the order of operators is significant here. In each term, commutators should appear only on the far left, powers of  $\Lambda$  only on the far right. Higher-order commutators should have  $[\Lambda, \Omega]$  only on the far right.
4. A useful and important property of Hermitian operators is the following: If  $\Omega$  and  $\Lambda$  are two commuting Hermitian operators, there exists a basis of common eigenvectors that diagonalizes both operators.

Read Shankar pp. 43–46, then do Ex. 1.8.10 on p. 46.

5. Merzbacher Ex. 10.6: Show that if Hermitian operators  $\Omega$  and  $\Lambda$  commute,  $\Omega$  has no nonvanishing matrix elements between eigenstates corresponding to different eigenvalues of  $\Lambda$ .
6. Adapted from Ballentine Problem 1.10: Consider the Hilbert space  $\mathcal{H}$  consisting of all complex-valued functions  $f(x)$  such that

$$\int_{-\infty}^{\infty} dx |f(x)|^2 < \infty,$$

and the reduced Hilbert space  $\mathcal{H}_{\mathcal{R}}$  containing those elements of  $\mathcal{H}$  that vanish faster than any power of  $|x|$  for  $|x| \rightarrow \infty$ . Let  $\mathcal{H}^{\mathcal{D}}$  and  $\mathcal{H}_{\mathcal{R}}^{\mathcal{D}}$  be the duals of these two vector spaces.

State which (if any) of the four vector spaces  $\mathcal{H}$ ,  $\mathcal{H}^{\mathcal{D}}$ ,  $\mathcal{H}_{\mathcal{R}}$ , and  $\mathcal{H}_{\mathcal{R}}^{\mathcal{D}}$  each of the following functions belongs to: (a)  $\cos ax$ ; (b)  $(\cos x)/x$ ; (c)  $\exp(iax)$ ; (d)  $\exp(-a^2|x|)$ ; (e)  $\exp(a^2x^2)$ ; (f)  $1/(a^2 + x^2)$ ; (g)  $\tan ax$ . You may assume that  $a$  is a real scalar.