

PHY 6645 Fall 2001 – Homework 3

Due at the start of class on Thursday, September 27. No credit will be available for solutions submitted after 4 p.m. on Friday, September 28.

Answer all questions. To gain full credit you should explain your reasoning and show all working. Please write neatly and remember to include your name on the front page of your answers.

1. Consider four possible quantizations of the classical dynamical variable $\omega = x^2 p^2$: $\Omega_1 = X^2 P^2$, $\Omega_2 = P^2 X^2$, $\Omega_3 = \frac{1}{2}(X^2 P^2 + P^2 X^2)$, and $\Omega_4 = \frac{1}{4}(XP + PX)^2$. Use the commutation relation between X and P to simplify as much as possible
 - (a) $\Omega_1 - \Omega_2$,
 - (b) $\Omega_1 - \Omega_3$,
 - (c) $\Omega_2 - \Omega_4$.

Here, simplification means expressing the result in terms of the lowest possible powers of the operators X and P . Also, you should express each term in such a way that any residual X operators are placed to the left of any P operators.

2. Based on Ballentine Problem 2.9: Let $R = \begin{pmatrix} 6 & -2 \\ -2 & 9 \end{pmatrix}$ be a representation of the operator R corresponding to some dynamical variable r , and $|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ be an arbitrary, normalized state vector (with $|a|^2 + |b|^2 = 1$). Suppose that $f(R)$ is some function of R . It is possible to calculate $\langle f(R) \rangle$ in two ways: (i) Evaluate $\langle f(R) \rangle = \langle \psi | f(R) | \psi \rangle$ directly. (ii) Find the eigenvalues and eigenvectors of R , $R|r_n\rangle = r_n|r_n\rangle$, expand the state vector as a linear combination of the eigenvectors, $|\psi\rangle = c_1|r_1\rangle + c_2|r_2\rangle$, and evaluate $\langle f(R) \rangle = f(r_1)|c_1|^2 + f(r_2)|c_2|^2$.
 - (a) Use both methods (i) and (ii) to evaluate $\langle R \rangle$.
 - (b) Use both methods (i) and (ii) to evaluate $\langle R^2 \rangle$.
 - (c) Find the uncertainty ΔR .
3. Shankar Exercise 4.2.1. As Shankar says, this exercise is very important. Therefore, it will be graded even though the answers are given in the book.