

PHY 6645 Fall 2001 – Homework 5

**Due at the start of class on Thursday, October 18. No credit will be available for solutions submitted after 4 p.m. on Friday, October 19.**

*Answer all questions. To gain full credit you should explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.*

1. Based on Merzbacher Section 3.6 and Sakurai Problem 2.6: Consider a particle in three dimensions whose Hamiltonian is  $H = \mathbf{P}^2/(2m) + V(\mathbf{R})$ .

(a) By calculating  $[\mathbf{R} \cdot \mathbf{P}, H]$ , show that

$$\frac{d}{dt} \langle \mathbf{R} \cdot \mathbf{P} \rangle = \left\langle \frac{\mathbf{P}^2}{m} \right\rangle - \langle \mathbf{R} \cdot \nabla V \rangle. \quad (1)$$

- (b) Show that, despite the noncommutativity of  $\mathbf{R}$  and  $\mathbf{P}$ , reversing the order of these operators on the left-hand side of Eq. (1) has no effect on the right-hand side.
- (c) Under what conditions will the left-hand side of Eq. (1) vanish? [If this happens, the equation can be identified with the *virial theorem* of classical mechanics:  $\langle \mathbf{p}^2/m \rangle = \langle \mathbf{r} \cdot \nabla V \rangle$ , where  $\langle x \rangle = \lim_{\tau \rightarrow \infty} \tau^{-1} \int_0^\tau dt x(t)$ .]

2. Shankar Exercise 7.3.5.

3. Shankar Exercise 7.3.6.

4. Merzbacher Problem 5.4: For the energy eigenstates with  $n = 0, 1$ , and  $2$ , compute the probability that the coordinate  $x$  of a harmonic oscillator in state  $n$  has a value greater than the amplitude of a classical harmonic oscillator of the same energy.

5. For each of the eigenstates  $n$  of the harmonic oscillator listed below, generate a single graph, with  $y$  as the horizontal axis (extending over the range  $0 \leq y \leq y_{\max}$ ), plotting vertically three quantities (clearly labeled): (i)  $\psi_n(y)$ ; (ii)  $P_{\text{box,QM}}(y)$ , the probability of finding the oscillator inside a box of width  $\Delta y = 1$  centered on the dimensionless coordinate  $y$ ; and (iii)  $P_{\text{box,CM}}(y)$ , the corresponding probability for a classical harmonic oscillator of the same energy. You should use the same vertical scale for (ii) and (iii), but feel free to use a different vertical scale for (i) if it makes the plot clearer.

Generate such a graph for (a)  $n = 0$ ,  $y_{\max} = 4$ ; (b)  $n = 1$ ,  $y_{\max} = 4$ ; (c)  $n = 10$ ,  $y_{\max} = 7$ ; (d)  $n = 11$ ,  $y_{\max} = 7$ .

To answer this question, you will need to produce some sort of computer program. You may use any general-purpose programming language, or a higher-level programming environment such as Mathematica, Matlab, or Maple. You may choose to represent the wave function as a set of numerical values calculated at a sufficiently fine grid of  $y$  points, or as an algebraic function of  $y$  which you can manipulate symbolically. The main restrictions are that you should develop your own solution to the problem, not merely copy someone else's, and you should not use any predefined software for generating the harmonic oscillator wave functions  $\psi_n(y)$ . (It is acceptable to use standard library functions for the Hermite polynomials.)